Guidance of an Autonomous Agent for Coverage Applications using Range Only Measurement

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This paper addresses the problem of guiding an autonomous agent to cover a region surrounding a target. It is assumed that only range information is available to the agent. The target point is considered to be stationary. The region of interest can be changed by changing the initial conditions or the system parameters. In this work, we analyse the effect of the initial conditions only. We propose a simple control law which can be easily implemented. The analytical results obtained here are validated through simulation. The autonomous agent could be a ground or an aerial vehicle depending on the application. For surveillance applications, the target can be a landmark, where the agent can monitor the region surrounding the landmark. Other applications can be sprinkling fertilizers, watering a farm, etc. where a pre-defined area needs to be covered.

I. Introduction

THE coverage problem refers to the problem of guiding an autonomous agent to cover an unknown or predefined area. Several techniques have been proposed to achieve coverage, which vary depending on the application. There are several algorithms which are used to cover unknown areas. For example, a lot of work has been done on map based coverage techniques (see [1], [2], [3], [4] and references therein) which work by dividing the area into cells and then assigning values to the cells based on the presence of obstacles or free space. Wong et al.¹ used topological maps to solve the coverage problem and their results are verified by simulation tests which show that over 99% of the surface is covered. The topological maps, represent the environment as graphs where landmarks are nodes, and edges represent the connectivity between the landmarks. The algorithm achieves coverage with single mobile agent. Zelinsky et al.² also use maps for coverage but with a different approach. The solution ensures complete coverage by the use of a distance transform path planning methodology. The solution was simulated and implemented on an autonomous robot called Yamabico to give satisfactory performance. Stachniss et al.³ introduced the concept of coverage maps where each cell of a given grid corresponds to the amount of a cell which is covered by an obstacle; the coverage maps are improvement of occupancy grids which are based on the assumption that each cell is either occupied or free. The model presented in the paper allows updation of the coverage maps upon input obtained from sensors. Rutishauser et al.⁴ solve the problem of collaborative coverage using a swarm of networked miniature robots again by the use of grid based methods. For a multi-agent system, the problem of coverage, addressed by Batavia et al.,⁵ has been solved with high accuracy for a semi-structured environment. The approach has the advantage of an operator driving the outline of a desired coverage area as input to a coverage generation algorithm. Acar et al.⁶ have solved the coverage problem by the decomposition of the environment, by the use of voronoi diagrams.

Surveillance based systems, discussed in [7], [8] and [9], use wireless sensor networks to solve the coverage problem. The sensor nodes are capable of behaving autonomously. In [7] and [9], the coverage achieved is average, but more emphasis is laid on the placement of sensors for effective coverage; Dhillon et al.⁷ have based their work on fixed sensor nodes. Lai et al.⁸ discuss surveillance systems based on wireless sensor networks. In [8], the deployed sensors are divided into disjoint subsets of sensors, or sensor covers, such that each sensor cover can cover all targets and work in turns. A surveillance system based on wireless

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Figure 1: Coverage obtained by an Autonomous Agent

Figure 2: Engagement geometry between the agent and the target point

sensor networks has been discussed by Yan et al.¹⁰ The system provides a differentiated surveillance system which deploys sensors nodes in appropriate areas depending on the degree of surveillance required. But, the wireless sensor networks suffer from the problem of energy requirements which increase with the number of sensor nodes used in the network.

The problem discussed in this paper is formulated as follows. The autonomous agent is modeled as an unicycle. We assume, the region to be covered, is circular or annular around the target as shown in Fig. 3. The agent uses only distance information to cover the annular region. The annular region is specified by R_{min} and R_{max} . It is shown that R_{min} and R_{max} depend on the initial conditions. Using this, we have derived:

- The R_{min} and R_{max} , given any initial conditions.
- The initial conditions necessary to achieve a given R_{min} or R_{max} .

This paper is organized as follows. In Section II, we define the problem and analyze it in Section III. Section IV elaborates the analysis further for a particular situation. Simulations results are presented in Section V and Section VI concludes the paper and discusses the areas on which further work can be done in future.

II. Problem Statement

The paper addresses the coverage problem as shown in Fig. 3. Let the target be at (x_T, y_T) and is stationary at any instant $t > 0$, position of agent is given by $(x(t), y(t))$, as shown in Fig. 2. The kinematics of agent is as shown below:

$$\dot{x}(t) = v \cos \alpha(t) \quad (1)$$

$$\dot{y}(t) = v \sin \alpha(t) \quad (2)$$

$$\dot{\alpha}(t) = u(t) \quad (3)$$

where v is the linear velocity, which is a constant for our case, $\alpha(t)$ is the heading angle and $u(t)$ is the control input. In Fig. 2, $r(t)$ is the line-of-sight(LOS) distance and $\theta(t)$ is the LOS angle. Since only range measurement is feasible, agent knows only instantaneous $r(t)$ and not $\theta(t)$. Let VT denote a virtual point, called the virtual target which is at a distance $\rho r(t)$ from the target along the LOS, where $\rho \in (0, 1)$. In the control law, the lateral acceleration of the autonomous agent is made proportional to the distance between the virtual target and the agent, given by $\left((x_{VT} - x(t))^2 + (y_{VT} - y(t))^2\right)^{1/2} = (1 - \rho)r(t)$. So, the algorithm is defined in terms of $a_M(t)$ as:

$$a_M(t) = h(1 - \rho)r(t) \quad (4)$$

$$\dot{\alpha}(t) = \frac{a_M(t)}{v} = kr(t) \quad (5)$$

where h is the constant of proportionality between a_M and the distance, $(1 - \rho)r$, and $k = \frac{h(1-\rho)}{v}$. The parameter h is a system gain. The concept of the virtual target increases the flexibility of the algorithm

Figure 3: Region to be covered

Figure 4: (v_θ, v_r) plot

as we get another parameter ρ to be varied as needed. The algorithm generates various coverage patterns surrounding the stationary target. One of the patterns generated is shown in Fig. 1. In the next section, the control algorithm is analysed for area coverage applications.

III. Analysis

Consider the Fig. 2, the LOS between the agent and the target can be characterised as follows:

$$v_r = \dot{r} = -v \cos(\alpha - \theta) \quad (6)$$

$$v_\theta = r\dot{\theta} = -v \sin(\alpha - \theta) \quad (7)$$

Let, $\phi = \alpha - \theta$. From eqns. (5)-(7), we get,

$$\dot{\phi} = \dot{\alpha} - \dot{\theta} = kr + \frac{v \sin \phi}{r} \quad (8)$$

From eqns. (6) and (8),

$$(kr^2 - v_\theta)dr + vr \cos(\phi)d\phi = 0 \quad (9)$$

Solving the integration, we establish a relationship between r and ϕ as,

$$vr \sin \phi + \frac{kr^3}{3} = vr_0 \sin \phi_0 + \frac{kr_0^3}{3} \quad (10)$$

In the above equation, r_0 and ϕ_0 are the initial conditions. We define a variable K which contains all the terms corresponding to the initial conditions, such that,

$$K = \left(\frac{3vr_0 \sin(\phi_0)}{k} + r_0^3 \right) \quad (11)$$

Hence, Eqn. (10) can be re-written as:

$$r^3 + \frac{3vr \sin \phi}{k} - K = 0 \quad (12)$$

By setting Eqn. (6) to zero, we observe that r can take its maximum and minimum values that is R_{min} and R_{max} when ϕ equals either $\frac{3\pi}{2}$ or $\frac{\pi}{2}$. Next we look into how the kinematics varies in the (v_θ, v_r) plane. Upon squaring and adding Eqn. (6) and Eqn. (7), we find that the instantaneous v_θ and v_r lie on a circle in the (v_θ, v_r) plane as shown in Fig. 4. By the use of Eqns. (6) and (7), we can calculate the value of ϕ corresponding to each set of instantaneous v_r and v_θ . Point A corresponds to $(v_\theta, v_r) = (v, 0)$, thus, $\phi = \frac{3\pi}{2}$. Similarly, we get that at B, $\phi = \pi$, at C, $\phi = \frac{\pi}{2}$ and at D, $\phi = 0$. Hence, the clockwise movement about the circle shown in Fig. 4 gives the direction of increasing values ϕ . Combining Eqns. (8) and (12), we get,

$$\frac{d\phi}{dt} = \frac{k(K + 2r^3)}{3r^2} \quad (13)$$

Figure 5: Engagement trajectory 1 in (v_θ, v_r) space

Figure 6: Engagement trajectory 2 in (v_θ, v_r) space

The rate of change of ϕ with respect to time varies depending on the sign of K . Considering the two cases separately, we have:

Case 1: $K > 0$

The above equation indicates that for $K > 0$ we get $\dot{\phi} > 0$. This corresponds to clockwise movement in the (v_θ, v_r) plane as shown in Fig. 5. Now, $v_r > 0$ in the path $C - B - A$ which means instantaneous r increases in this path upto point A and thereafter it decreases as $v_r < 0$ in the path $A - D - C$. Hence, R_{max} occurs at $A(\phi = \frac{3\pi}{2})$ and R_{min} occurs at $C(\phi = \frac{\pi}{2})$.

Case 2: $K \leq 0$

v_r and v_θ are continuous functions of ϕ . By re-arranging Eqn. (12), we get $\sin \phi = \frac{k(K-r^3)}{3vr}$. Hence, from Eqn. (7) for all values of $K \leq 0$, $v_\theta \geq 0$. This indicates that the engagement geometry can exist only in the first and fourth quadrants in the (v_θ, v_r) plane. So, the only feasible inflexion point is $\phi = \frac{3\pi}{2}$. Hence, considering the continuity of the v_θ and v_r functions, we conclude that the (v_θ, v_r) plot oscillates about $\phi = \frac{3\pi}{2}$ as shown in Fig. 6. Now, $v_r > 0$ in the path $A - B - A$ which means instantaneous r increases in this path till the second time it reaches point A . Thereafter, it decreases as $v_r < 0$ in the path $A - C - A$ till it comes to the point A for the second time. Hence, in this case, both R_{max} and R_{min} occur at $A(\phi = \frac{3\pi}{2})$.

Thus, by using the coverage control algorithm given in Eqn. (5), we get two types of engagement trajectories in (v_r, v_θ) plane, depending on K or the initial conditions (r_0, ϕ_0) . By differentiating Eqns. (6) and (7), we get:

$$\dot{v}_r = -v_\theta \dot{\phi} \quad (14)$$

$$\dot{v}_\theta = v_r \dot{\phi} \quad (15)$$

At the points B and C shown in Fig. 6, $\dot{v}_r$ and $\dot{v}_\theta$ become zero simultaneously, which implies $\dot{\phi} = 0$. Using this condition in Eqn. (13), we get the following result for the instantaneous value of r :

$$r = \left(\frac{K'}{2} \right)^{\frac{1}{3}} \quad (16)$$

where $K' = (-K)$. Substituting eqn. (16) in eqn. (12) and further solving, we get an expression of instantaneous ϕ as,

$$\sin \phi = \frac{-k}{v} \left(\frac{K'^2}{4} \right)^{\frac{1}{3}} \quad (17)$$

Eqn. (17) gives a bound on $K' = (-K)$ as sine function can take values between -1 to 1. Hence, the minimum value of K can be obtained by setting $\sin \phi = -1$, which gives $K = -K' = -2\sqrt{\frac{v^3}{k^3}}$. Let us denote

the lower value of K by: $-K_{br} = -2\sqrt{\frac{v^3}{k^3}}$. Thus, coverage becomes feasible for negative values of K only if $-K_{br} \leq K \leq 0$. Next we address two problems: If we are given an initial condition, what will be the coverage area and if we are given an area to be covered, what should be the initial conditions to achieve the coverage?

III.A. Achieving coverage for a specified r_0 and ϕ_0

Given an initial condition (r_0, ϕ_0) , we have the following result:

Theorem 1: Given a set of initial conditions (r_0, ϕ_0) for an agent with kinematics (1)-(3) and a control law given by (5), coverage will be achieved for a region given by the maximum radius R_{max} and minimum radius R_{min} which are given as:

$$R_{max} = az_1 + bz_2 \quad (18)$$

$$R_{min} = az_3 + bz_4 \quad (19)$$

where

$$a = \begin{cases} \frac{-1+i\sqrt{3}}{2}, & \text{if } -K_{br} \leq K \leq 0 \\ 1, & \text{if } 0 < K < \infty \end{cases} \quad z_1 = -\left(-\frac{K}{2} + \sqrt{\frac{K^2}{4} - \frac{v^3}{k^3}}\right)^{\frac{1}{3}}$$

$$b = \begin{cases} \frac{-1-i\sqrt{3}}{2}, & \text{if } -K_{br} \leq K \leq 0 \\ 1, & \text{if } 0 < K < \infty \end{cases} \quad z_2 = -\left(-\frac{K}{2} - \sqrt{\frac{K^2}{4} - \frac{v^3}{k^3}}\right)^{\frac{1}{3}}$$

$$z_3 = \begin{cases} z_2 & \text{if } -K_{br} \leq K \leq 0 \\ -\left(-\frac{K}{2} + \sqrt{\frac{K^2}{4} + \frac{v^3}{k^3}}\right)^{\frac{1}{3}} & \text{if } 0 < K < \infty \end{cases}$$

$$z_4 = \begin{cases} z_1 & \text{if } -K_{br} \leq K \leq 0 \\ -\left(-\frac{K}{2} - \sqrt{\frac{K^2}{4} + \frac{v^3}{k^3}}\right)^{\frac{1}{3}} & \text{if } 0 < K < \infty. \end{cases}$$

Proof: The calculation of R_{max} and R_{min} are done on the basis of the (v_θ, v_r) plots. Given the Figs. 5 and 6, we have derived that R_{min} and R_{max} occur at $\phi = \pi/2$ and $\phi = 3\pi/2$ while $K \in (-\infty, \infty)$. Hence, we have the four cases:

- Case 1 : $K > 0, \quad \phi = 3\pi/2$
- Case 2 : $K > 0, \quad \phi = \pi/2$
- Case 3 : $K \leq 0, \quad \phi = 3\pi/2$
- Case 4 : $K \leq 0, \quad \phi = \pi/2$

Case 1: $\phi = 3\pi/2$ and $K > 0$

The (v_θ, v_r) plot of Figure 5 shows that instantaneous r attains a maximum for $\phi = 3\pi/2$. Substituting the value of ϕ in Eqn. (12), we get a cubic equation in r . The nature roots of Eqn. (12) can be obtained using root locus plot. In order to do that we re-arranged Eqn. (12) to get the Evans form as $(1 + \frac{-K}{r^3 - \frac{3vr}{k}} = 0)$. Hence, we get the root locus plot shown Fig. 7. It shows that for $0 < K < \infty$, the equation has one positive solution. From (v_θ, v_r) plot we find that this root corresponds to the maximum value of instantaneous r . Solving Eqn. (12), we get the value of the root as:

Figure 7: Root Locus for positive K and $\phi = 3\pi/2$

Figure 8: Root Locus for positive K and $\phi = \pi/2$

$$R_{max} = - \left(-\frac{K}{2} + \sqrt{\frac{K^2}{4} - \frac{v^3}{k^3}} \right)^{\frac{1}{3}} - \left(-\frac{K}{2} - \sqrt{\frac{K^2}{4} - \frac{v^3}{k^3}} \right)^{\frac{1}{3}} \quad (20)$$

Case 2: $\phi = \pi/2$ and $K > 0$

Proceeding the same way as for the previous case, we conclude that for $K > 0$, $\phi = \frac{\pi}{2}$ corresponds to the minimum value of the instantaneous r . Hence, by substituting this condition in Eqn. (12), we get a cubic equation in r . To analyse the behaviour of the roots, we re-arrange Eqn. (12), to get the Evan's form as $(1 + \frac{-K}{r^3 + \frac{3vr}{k}} = 0)$. The root locus plot of the equation is given in Fig. 8. It shows that for $0 < K < \infty$, the equation has one positive solution which corresponds to the minimum value of instantaneous r . Solving Eqn. (12), we get the value of the root as:

$$R_{min} = - \left(-\frac{K}{2} + \sqrt{\frac{K^2}{4} + \frac{v^3}{k^3}} \right)^{\frac{1}{3}} - \left(-\frac{K}{2} - \sqrt{\frac{K^2}{4} + \frac{v^3}{k^3}} \right)^{\frac{1}{3}} \quad (21)$$

Case 3: $\phi = 3\pi/2$ and $K \leq 0$

The (v_θ, v_r) plot of Fig. (6) shows that for $\phi = 3\pi/2$, the instantaneous r attains both its maximum and minimum values in an alternate manner. Substituting the value of ϕ in Eqn. (12), we get the cubic equation in terms of r . To generate the root locus, we re-arranged Eqn. (12) to get the Evans form as $(1 + \frac{K'}{r^3 - \frac{3vr}{k}} = 0)$, where $K' = -K$. The root locus plot, shown in Fig. 9, shows that for $0 \leq K' \leq K_{br}$, the equation yields two positive roots indicating R_{max} and R_{min} . But, beyond $K' = K_{br}$ which corresponds to the breakaway value of K' , the two positive roots become imaginary. Hence, no value of K' greater than K_{br} gives a feasible solution. At the breakaway value of $K' = K_{br} = 2(\frac{v}{k})^{\frac{3}{2}}$, R_{max} and R_{min} become equal. Now, before solving the eqn. (12), we define two terms $z_1 = - \left(-\frac{K}{2} + \sqrt{\frac{K^2}{4} - \frac{v^3}{k^3}} \right)^{\frac{1}{3}}$ and $z_2 = - \left(-\frac{K}{2} - \sqrt{\frac{K^2}{4} - \frac{v^3}{k^3}} \right)^{\frac{1}{3}}$. The instantaneous value of r corresponding to its maximum and minimum values are given by,

$$R_{max} = \left(\frac{-1 + i\sqrt{3}}{2} \right) z_1 + \left(\frac{-1 - i\sqrt{3}}{2} \right) z_2 \quad (22)$$

$$R_{min} = \left(\frac{-1 - i\sqrt{3}}{2} \right) z_1 + \left(\frac{-1 + i\sqrt{3}}{2} \right) z_2 \quad (23)$$

Figure 9: Root Locus for negative K and $\phi = 3\pi/2$

Figure 10: Root Locus for negative K and $\phi = \pi/2$

Case 4: $\phi = \pi/2$ and $K \leq 0$

We substituted these values in Eqn. (12) to get the Evans form as $(1 + \frac{K'}{r^3 + \frac{3vr}{k}} = 0)$. By using the Evans form, we obtained the root locus plot as shown in Fig. 10. It shows that, no feasible roots exist. Hence, coverage cannot be attained for this configuration.

Hence, we can conclude from the four cases that given an initial condition, the maximum and minimum radii of the annular region that can be covered by the agent is as given in Eqns. (20), (21), (22) and (23). $\square$

III.B. Achieving coverage with desired R_{max} and R_{min}

In the previous section, we saw that for any initial condition, we can calculate the value of K . Using that K value, we can obtain the maximum and minimum value that the instantaneous r can take. Now, if the user specifies the R_{max} and R_{min} values, it is possible to generate the initial conditions a priori by performing some calculations. But, as for the algorithm, R_{max} and R_{min} are always dependent on each other. So, once R_{max} is specified by the user, then, R_{min} can take only specific values. The relationship existing between R_{max} and R_{min} can be established as follows: For any K satisfying $-K_{br} \leq K \leq 0$, $\phi = \frac{3\pi}{2}$ gives both R_{max} and R_{min} . So, replacing r by R_{max} or R_{min} and the corresponding value of $\phi = \frac{3\pi}{2}$ in Eqn. (12) gives, $K = R_{min}^3 - \frac{3vR_{min}}{k} = R_{max}^3 - \frac{3vR_{max}}{k}$. For any value of K such that $0 < K < \infty$, by performing similar calculations, we get, $K = R_{min}^3 + \frac{3vR_{min}}{k} = R_{max}^3 - \frac{3vR_{max}}{k}$. Hence, given desired R_{max} or R_{min} , then the parameter K can be evaluated. Eqn. (11) can be re-arranged to get,

$$\sin \phi_0 = \frac{k(K - r_0^3)}{3vr_0} \quad (24)$$

Case 1: $0 < K < \infty$

From the (v_θ, v_r) , shown in Figure 5, we know that for $0 < K < \infty$, $-v < v_\theta < v$ which implies $-1 < -\sin \phi_0 < 1$ and $\phi_0 \in (0, 2\pi)$. Hence, by using (24), we get the range of feasible values of r_0 given by,

$$-1 < \frac{k(K - r_0^3)}{3vr_0} < 1 \quad (25)$$

Case 2: $-K_{br} \leq K \leq 0$

For this case, we get the (v_θ, v_r) as shown in Figure 6. This indicates $0 \leq v_\theta \leq 1$ which implies $-1 \leq -\sin \phi_0 \leq 0$ and $\pi \in [\pi, 2\pi]$. So, combining this result with Eqn. (24), we get,

$$-1 \leq \frac{k(K - r_0^3)}{3vr_0} \leq 0 \quad (26)$$

Hence, for any $r_0 = \bar{r}_0$, chosen from the range of values obtained from either Eqn. (25) or eqn. (26), the appropriate value of $\phi_0 = \bar{\phi}_0$ can be calculated from Eqn.(24) i.e. $\sin \bar{\phi}_0 = \frac{k(K - \bar{r}_0^3)}{3v\bar{r}_0}$. Hence, for each value

Figure 11: $(r_0, \sin \phi_0)$ for a range of K values

of K , we get a range of feasible values of $(r_0, \sin \phi_0)$.

If we plot the feasible values of $(r_0, \sin \phi_0)$ for a range of values of K , we get the plot shown in Fig. 11. For $K \leq 0$, a particular $\sin \phi_0$ value can map to two values of r_0 . For $K \geq 0$, a particular $\sin \phi_0$ value can map to one value of r_0 . For every point in the $(r_0, \sin \phi_0)$ space with $r_0 > 0$, we get a feasible value of K . Hence, coverage is ensured for every feasible point in the $(r_0, \sin \phi_0)$ space. The method illustrated in this section can be summarised by the algorithm which follows: Given R_{max} , then the initial conditions necessary to achieve coverage of the region of interest are given by:

1. $K = R_{max}^3 - \frac{3vR_{max}}{k}$
2. If $K > 0$, then $R_{min}^3 + \frac{3vR_{min}}{k} - K = 0$, else $R_{min}^3 - \frac{3vR_{min}}{k} - K = 0$. Solution of the appropriate equation gives R_{min} .
3. If $K > 0$, then choose any r_0 such that $-1 < \frac{k(K-r_0^3)}{3vr_0} < 1$, else choose r_0 such that $-1 \leq \frac{k(K-r_0^3)}{3vr_0} \leq 0$.
4. The value of ϕ_0 is given by, $\phi_0 = \arcsin(\frac{k(K-r_0^3)}{3vr_0})$.

Given R_{min} , then the initial conditions necessary to achieve coverage of the region of interest are given by:

1. $K = R_{min}^3 - \frac{3vR_{max}}{k}$ or $K = R_{min}^3 + \frac{3vR_{max}}{k}$.
2. Then $R_{max}^3 - \frac{3vR_{max}}{k} - K = 0$. Solution of the equation gives R_{max} .
3. If $K > 0$, then choose any r_0 such that $-1 < \frac{k(K-r_0^3)}{3vr_0} < 1$, else choose r_0 such that $-1 \leq \frac{k(K-r_0^3)}{3vr_0} \leq 0$.
4. The value of ϕ_0 is given by, $\phi_0 = \arcsin(\frac{k(K-r_0^3)}{3vr_0})$.

Once r_0 is obtained, any starting point, (x_0, y_0) , in the cartesian coordinate system can be chosen such that $r_0^2 = x_0^2 + y_0^2$. Then, θ_0 is calculated by $\theta = \arctan(\frac{y_0}{x_0})$. Now, the ϕ_0 , obtained using the algorithm, is used to generate the heading angle of the agent given by, $\alpha_0 = (\theta_0 + \phi_0)$. Thus, the initial coordinates obtained are given by: (x_0, y_0, α_0) .

III.C. Critical Initial Position

Critical Initial Position refers to that value of (r_0, ϕ_0) for which R_{max} and R_{min} are equal. This makes the agent move on a circular path around the stationary target. Also, it is interesting to note that for these initial coordinates, we get $r_0 = R_{max} = R_{min}$.

Theorem 2: For $r_0 = \sqrt{\frac{v}{k}}$ and $\phi_0 = 3\pi/2$, the trajectory of an agent with kinematics (1)-(3) and a control law given by (5), is a circle of radius $\sqrt{\frac{v}{k}}$.

Proof: Given the initial conditions $r_0 = \sqrt{\frac{v}{k}}$ and $\phi_0 = \frac{3\pi}{2}$. From (11), we get $K = -2\sqrt{\frac{v^3}{k^3}}$. In order

to calculate the R_{max} and R_{min} using the K and ϕ_0 values, we refer to *Theorem 1*. Since $K \in [-K_{br}, 0]$, from case III of theorem, we get that $K = -2\sqrt{\frac{v^3}{k^3}}$ corresponds to the breakaway point of the root locus given in Fig. 9. At the breakaway point, two roots of the Eqn. (12) are real and equal and the third root is imaginary, as shown in Fig. 9. Hence, by using Eqns. (22) and (23) for the given initial conditions, we get R_{max} and R_{min} as:

$$R_{max} = R_{min} = \sqrt{\frac{v}{k}} \quad (27)$$

The given initial conditions, which make the agent circle around the stationary target, are referred to as the critical initial position, given by,

$$\begin{aligned} r_0 = r_{cr} &= \sqrt{\frac{v}{k}} \\ \sin \phi_0 = -1 &\Rightarrow \phi_0 = 3\pi/2 \end{aligned} \quad (28)$$

From (27) and (28), we conclude that $r_0 = R_{max} = R_{min}$. Hence, the autonomous agent circles around the target with a fixed radius of $\sqrt{\frac{v}{k}}$ for all time $t > 0$. $\square$

IV. Special case: Behaviour of the system when $\phi_0 = 3\pi/2$

The coverage algorithm, designed here, ensures coverage for all feasible values of initial radial distance, r_0 and ϕ_0 . But, the particular situation of $\phi_0 = 3\pi/2$ gives interesting results for R_{max} and R_{min} . Also, in the previous section, we saw that for $\phi_0 = 3\pi/2$ the agent revolves around the target with a radius $r = \sqrt{\frac{v}{k}}$. Now, we have $K = (r_0^3 - \frac{3vr_0}{k})$. Hence, for $0 \leq r_0 \leq \sqrt{\frac{3v}{k}}$, we get $K \leq 0$ and for $r_0 > \sqrt{\frac{3v}{k}}$, we get $K > 0$.

IV.A. Maximum & Minimum radii for any initial radial distance, r_0

In this section, we have derived the R_{max} and R_{min} for $\phi_0 = \frac{3\pi}{2}$. The R_{max} and R_{min} follow patterns as we vary the initial radial distance r_0 .

Case 1: $K \leq 0$

From the (v_θ, v_r) plot of Fig. 6, we get $\phi = \frac{3\pi}{2}$ corresponds to both R_{max} and R_{min} . Simplifying the eqns. (22) and (23) yields two positive roots, R_{max} and R_{min} :

$$r_1 = r_0 \quad (29)$$

$$r_2 = \frac{-r_0 + \sqrt{-3r_0^2 - 4\frac{12v}{k}}}{2} \quad (30)$$

Now, depending on r_0 , one of the roots becomes R_{max} and the other R_{min} . For initial radial distance less than $r_0 = r_{cr}$, $r = r_0$ becomes R_{min} and the other root becomes R_{max} which exactly reverses for $r_{cr} \leq r_0 \leq \sqrt{\frac{3v}{k}}$. So, the behaviour of the R_{max} and R_{min} vary in accordance to the initial radial distance from the target.

Case 2: $K > 0$

The (v_θ, v_r) plot of Figure 5 gives $\phi = \frac{3\pi}{2}$ indicates R_{max} and $\phi = \frac{\pi}{2}$ indicates R_{min} . Simplifying the Eqns. (20) and (21), we get the two positive roots as:

$$R_{max} = r_0 \quad (31)$$

$$R_{min} = -\left(\frac{K}{2} + \sqrt{\frac{K^2}{4} + \frac{v^3}{k^3}}\right)^{\frac{1}{3}} - \left(\frac{K}{2} - \sqrt{\frac{K^2}{4} + \frac{v^3}{k^3}}\right)^{\frac{1}{3}} \quad (32)$$

All the above results were verified from their second derivatives.

Figure 12: R_{max} for $\phi = 3\pi/2$, $0 < r_0 < \sqrt{\frac{v}{k}}$

Figure 13: R_{max} for $\phi = 3\pi/2$, $\sqrt{\frac{v}{k}} < r_0 < \sqrt{\frac{3v}{k}}$

Figure 14: R_{min} for $\phi = 3\pi/2$ and $0 < r_0 < \sqrt{\frac{v}{k}}$

Figure 15: R_{min} for $\phi = 3\pi/2$ and $\sqrt{\frac{v}{k}} < r_0 < \sqrt{\frac{3v}{k}}$

IV.B. Agent restricts itself within a stretch of radius $\sqrt{\frac{3v}{k}}$

Given, $\phi_0 = \frac{3\pi}{2}$, the trajectory of the agent has a peculiar behaviour: For $0 \leq r_0 \leq \sqrt{\frac{3v}{k}}$, the maximum radial distance that can be covered by the agent can reach a maximum of $R_{max} = \sqrt{\frac{3v}{k}}$ and never exceed it. In a similar manner, the minimum radius that the agent can reach is $R_{min} = 0$.

Corollary 1: Given $\phi_0 = \frac{3\pi}{2}$, for any initial position $r_0 \in [0, \sqrt{\frac{3v}{k}}]$, the trajectory of the agent is bound by a circle of radius $\sqrt{\frac{3v}{k}}$.

Proof: In the previous section, we derived the values of R_{max} and R_{min} for any arbitrary initial position r_0 . Eqns. (29) and (30) establish a relationship of R_{max} and R_{min} with r_0 . The nature of the variation R_{max} and R_{min} with respect to r_0 can be explained by plotting (29) and (30). Fig. 12 shows the variation of R_{max} for $r_0 \in [0, \sqrt{\frac{v}{k}}]$ and Fig. 13 shows the same for $r_0 \in [\sqrt{\frac{v}{k}}, \sqrt{\frac{3v}{k}}]$.

For $r_0 \in [0, \sqrt{\frac{v}{k}}]$, Fig. 14 shows variation of R_{min} with respect to r_0 and Fig. 15 shows variation of R_{min} for $r_0 \in [\sqrt{\frac{v}{k}}, \sqrt{\frac{3v}{k}}]$. From the plots, it is clear that $\max(R_{max}) = \sqrt{\frac{3v}{k}}$ and $\min(R_{min}) = 0$. This ensures that if the agent starts with $\phi_0 = 3\pi/2$, then for any value of r_0 , the agent will always remain within a circle of radius $r = \sqrt{\frac{3v}{k}}$. $\square$

For the particular case of $r_0 = 0$ and $\phi_0 = \frac{3\pi}{2}$, since $r_0 \in [0, \sqrt{\frac{v}{k}}]$ we refer to Eqn. (29), to get $R_{max} = \sqrt{\frac{3v}{k}}$ and $R_{min} = 0$. This makes the agent cover the entire area of the circle of radius $\sqrt{\frac{3v}{k}}$ centred around the stationary target.

V. Simulation Results

In this section, we validate the results obtained in sections III and IV through MATLAB simulations. The simulations were performed with the parameters $h = 0.01$, $\rho = 0.3$ and $v = 5$. The control algorithm developed in Eqn. (5) leads to formation of patterns around the target ensuring coverage of the area

Figure 16: Trajectory plot: $r_0 = 45$, $\phi_0 = \frac{110\pi}{180}$

Figure 17: r versus t : $r_0 = 45$, $\phi_0 = \frac{110\pi}{180}$

Figure 18: Range of r_0

surrounding the target.

V.A. Achieving coverage for a specified r_0 and ϕ_0

Let us consider the initial conditions of $r_0 = 45$ and $\phi_0 = \frac{110\pi}{180}$ which give $0 < K < \infty$ and hence achieve maximum and minimum radii as per Fig. 5. Now, by using *Theorem 1*, the maximum and minimum radii of area covered can be calculated using Eqns. (20) and (21) and are given as:

$$\begin{aligned} R_{max} &= 177.4053 \\ R_{min} &= 42.7959 \end{aligned} \tag{33}$$

The trajectory plot, for the given initial conditions, is as shown in Fig. 16. From the figure, it is verified that coverage occurs. Now, the R_{max} and R_{min} can be verified from the Fig. 17 which shows the variation of instantaneous radial distance r , generated using MATLAB simulation.

V.B. Achieving coverage with desired R_{max} and R_{min}

Let us consider a situation given by desired $R_{max} = 100$. Using $K = R_{max}^3 - \frac{3vR_{max}}{k}$, we proceed to calculate $K = -1500000$. Since, $-K_{br} \leq K \leq 0$, using the relationship $K = R_{min}^3 - \frac{3vR_{min}}{k}$, we get $R_{min} = 82.28757$. Now, $-K_{br} \leq K \leq 0$, hence, as given in section III.B, we get the restrictions on ϕ_0 . Substituting the value of K in Eqn. (26) gives a range of values of r_0 as shown in Fig.18. From Fig.18, a suitable value of $r_0 = 95$ is chosen. Using Eqn. (24), we get $\phi_0 = 263.0154$ degrees. Now, in simulations, using the above initial conditions, coverage is achieved, with desired $R_{max} = 100$. Hence, the result is verified by the simulation results shown in Fig. 20 and 19.

Figure 19: Trajectory plot: for $r_0 = 95$, $\phi_0 = 263.0154$

Figure 20: r versus t : $r_0 = 95$, $\phi_0 = 263.0154$

VI. Conclusion

In the paper, we presented a strategy that can make an autonomous agent cover the region surrounding a stationary target, using only range measurement. We derived the R_{max} and R_{min} values which characterise the region covered by the autonomous agent under the control law (5), given any initial conditions (r_0, ϕ_0) . Similarly, we derived the initial conditions necessary to cover the region-of-interest when R_{max} and R_{min} are given. We also showed that under certain initial conditions, the agent covers the area around the target by revolving around the stationary target on a circle of fixed radius. The radius of the circular trajectory depends on the fixed parameters involved in the system of equations and, hence, can be specified by the user. As the guidance strategy uses only range measurement to achieve coverage, the energy requirements of the entire system are minimised. In the paper, the effect of initial conditions on the pattern generation has been studied. The work can be extended to study the effect of system parameters, like ρ , on the patterns generated. Further work can also be done to calculate the time required to cover the area specified by the user.

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