Networked and distributed CPS: control under network constraints and networked transportation systems

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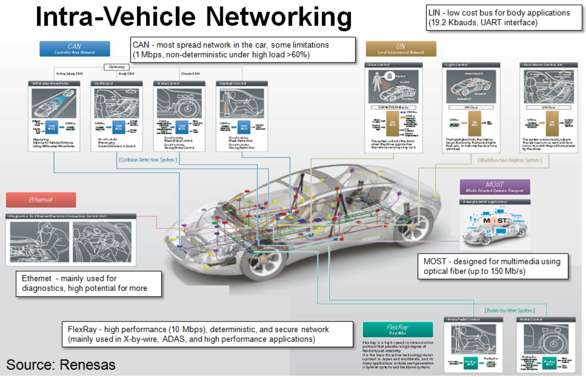
Cyber physical systems



- Hundreds of sensors, actuators and processors all communicating over a network; millions of lines computer code

- Vast geographical spread, thousands of nodes - hierarchical and distributed topologies

Cyber physical systems

- Integrated approach to the design of control, communication and computing components - Cyber Physical Systems (CPS)

Cyber physical systems

- Challenges: Constrained resources (energy, communication, computation), privacy and security . . .

Outline

1. Opportunistic state-triggered control
2. Differential privacy in CPS
3. Networked transportation systems
4. Summary & future research plans

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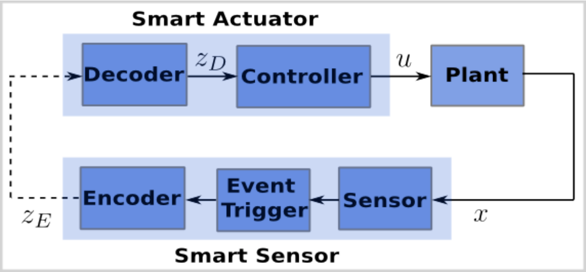
Networked control systems



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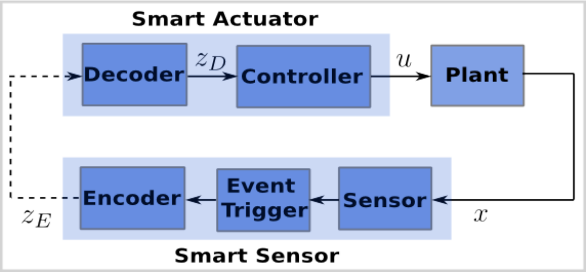
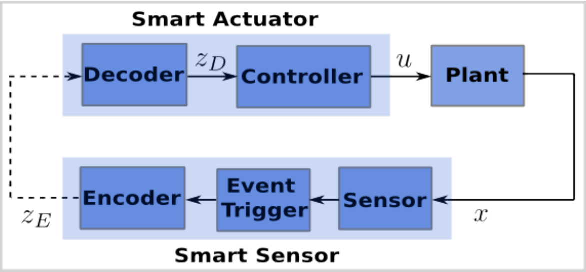


- **When to transmit:**
  Time-triggered strategies
  - The traditional approach to sampling
  - Usually the triggering is periodic
  - Novelty of the sensor data not important in the sampling decision

Networked control systems



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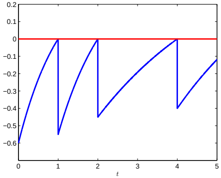
  - A trigger function implicitly determines transmission times
  - Trigger function encodes the control goal
  - Transmissions occur only when necessary
  - Better use of resources than time-triggered



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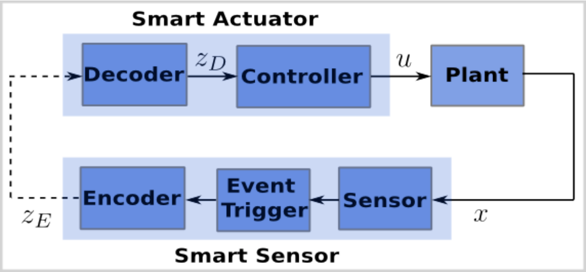

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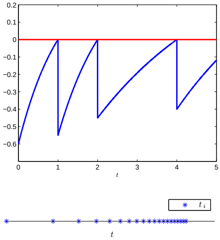
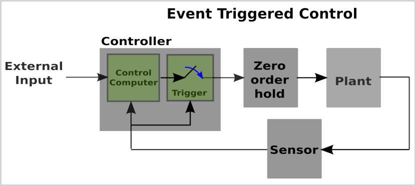
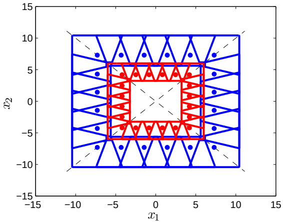
  - Need to ensure Zeno does not occur



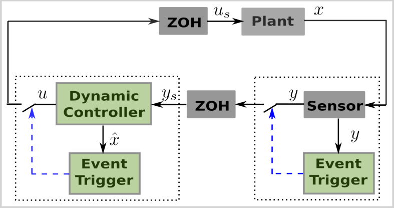
Event-triggered control under imperfect information



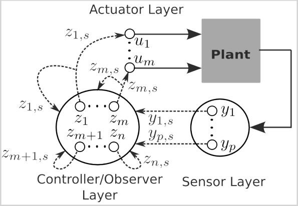
Online trajectory tracking



Quantization and event-triggering co-design



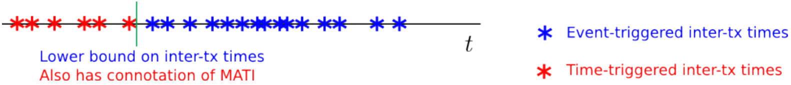
Dynamic output feedback control



Decentralized control

Ph.D. work

What is the case for event-triggered control?



MATI is a lower bound on inter-transmission times for an event-triggered implementation

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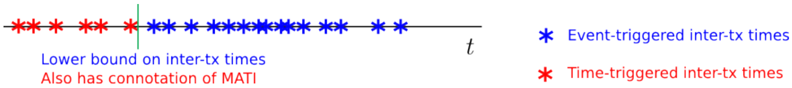

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- But what about the distribution or the average of the inter-transmission times?
- More generally, what is the average data rate?

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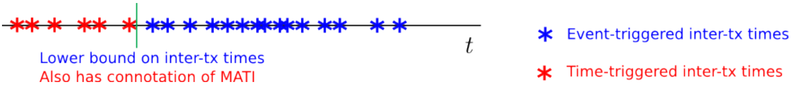

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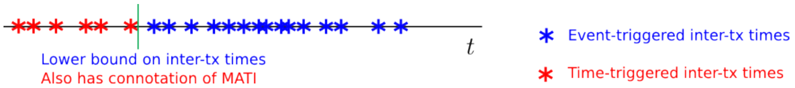

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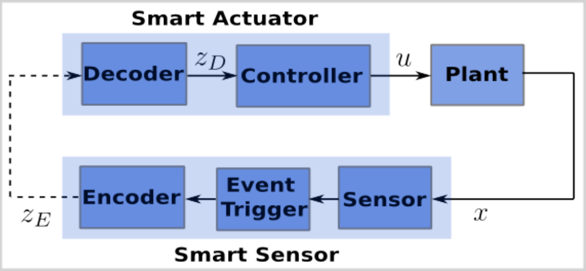
- But what about the distribution or the average of the inter-transmission times?
- More generally, what is the average data rate?
- These are open questions in general

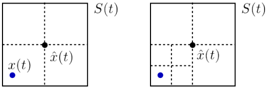
- Can we design controllers with analytically quantifiable data rate?
- Given a bound on the channel data capacity, what should the transmission policy be?

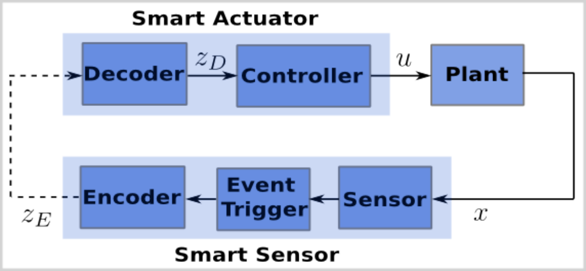
Networked control systems - what to transmit



Information-theory based data rate theorems

- Quite successful in the discrete-time setting
- Tight necessary and sufficient data rates for stabilization





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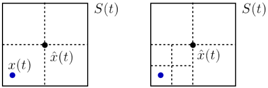

What about sufficient rates for specific performance (e.g. convergence rate)?

System description

Plant dynamics:

$\dot{x}(t) = Ax(t) + Bu(t) + v(t), \quad u(t) = K\hat{x}(t), \quad x(t) \in \mathbb{R}^n, \quad \|v(t)\|_2 \leq \nu,$
$\forall t \in [t_0, \infty]$

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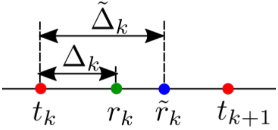
Communication model:



$\Delta_k \leq \Delta(t_k, p_k) \triangleq \frac{b_k}{R_a(t_k)} = \frac{p_k}{R(t_k)}$

# of bits transmitted at t_k is $b_k = np_k$

Can choose $\{t_k\}$, $\{p_k\}$, $\{\tilde{r}_k\}$

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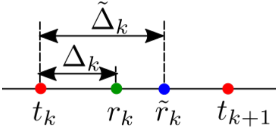

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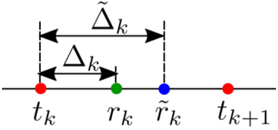

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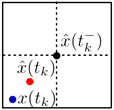
Dynamic controller jump: $\hat{x}(\tilde{r}_k) \triangleq q_k(x(t_k), \hat{x}(t_k^-))$



Encoding error: $x_e \triangleq x - \hat{x}$

Objective

Suppose $\bar{A} = A + BK$ is Hurwitz $\iff$ $P\bar{A} + \bar{A}^T P = -Q$

Lyapunov function: $x \mapsto V(x) = x^T P x$

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Desired performance function: $V_d(t) = (V_d(t_0) - V_0)e^{-\beta(t-t_0)} + V_0$

Performance objective: ensure $h_{\mathrm{pf}}(t) \triangleq \frac{V(x(t))}{V_d(t)} \leq 1$, for all $t \geq t_0$

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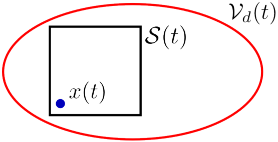
Design objective:

- Design event-triggered communication policy that is applicable to channels with time-varying rates and data capacity
- Recursively determine $\{t_k\}$, $\{p_k\}$ and $\{\tilde{r}_k\}$
- Ensure a uniform positive lower bound for $\{t_k - t_{k-1}\}_{k \in \mathbb{Z}_{>0}}$

Necessary data rate (non-state-triggered transmissions)



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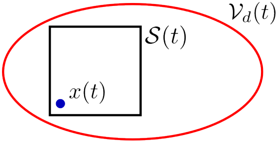

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Number of bits necessary to be transmitted between t_0 and t to meet the control goal:

$$\mathcal{B}(t, t_0) \geq \left(\operatorname{tr}(A) + \frac{n\beta}{2} \right) \log_2(e)(t - t_0) + \log_2 \left(\frac{\operatorname{vol}(\mathcal{S}(t_0))}{c_P (V_d(t_0))^{\frac{n}{2}}} \right)$$

$$R_{\mathrm{as}} \triangleq \lim_{t \to \infty} \frac{\mathcal{B}(t, t_0)}{t - t_0} \geq \left(\operatorname{tr}(A) + \frac{n\beta}{2} \right) \log_2(e)$$

Assuming all eigenvalues of A have real parts greater than $-\beta$.

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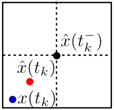
- If the decoder knows $d_e(t_0)$ s.t. $\|x_e(t_0)\|_\infty \le d_e(t_0)$
- Both encoder and decoder compute recursively:

$$d_e(t) \triangleq \|e^{A(t-t_k)}\|_\infty \delta_k, \; t \in [\tilde{r}_k, \tilde{r}_{k+1}), \; k \in \mathbb{Z}_{\ge 0}$$
$$\delta_{k+1} = \frac{1}{2^{p_{k+1}}} d_e(t_{k+1}).$$



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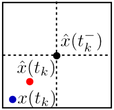

- Then, $\|x_e(t)\|_\infty \leq d_e(t)$, for all $t \geq t_0$

Control under bounded rate and capacity

Theorem

If

- $\bar{p}$ *is max. packet size*
- $R(t) \geq \frac{p}{T_M}, \ \forall t$

Control under bounded rate and capacity

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If

- $\bar{p}$ *is max. packet size*
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Then

- *Can design event-triggered* $\{t_k\}$, $\{p_k\}$, $\{\tilde{r}_k\}$
- *inter-transmission times have uniform positive lower bound*
- $V(x(t)) \leq V_d(t)$ *for* $t \geq t_0$
  (origin is exponentially practically stable if there is disturbance)

Upper bound on the sufficient data rate

Corollary (With disturbance)

Let $\bar{\theta} = \|A\|_\infty + \frac{\beta}{2}$. *For any* $k \in \mathbb{Z}_{>0}$,

$$\underline{p_k} \leq \log_2\left(\frac{e^{\bar{\theta}T_M}}{\rho_T(\tilde{b}(T_M, b(t_k^-), \epsilon(t_k^-)) - \alpha(T_M)}\right) + 1 + \log_2\left(\frac{e^{\bar{\theta}(t_k - t_0)}}{\prod_{j=1}^{k-1} 2^{p_j}}\epsilon(t_0) + \sum_{i=0}^{k-1}\prod_{j=i+1}^{k-1}\frac{e^{\bar{\theta}T_j}}{2^{p_j}}\alpha(T_i)\right).$$

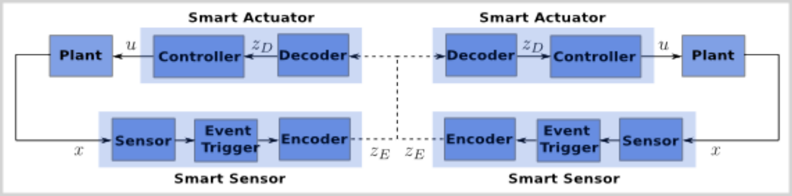
Corollary (No disturbance)

Let $\bar{\theta} = \|A\|_\infty + \frac{\beta}{2}$. *For any* $k \in \mathbb{Z}_{>0}$,

$$n\left(\underline{p_k} + \sum_{i=1}^{k-1} p_i\right) \leq n\left[\log_2\left(\frac{e^{\bar{\theta}T_M}}{\rho_T(\tilde{b}(T_M, b(t_k^-), \epsilon(t_k^-)))}\right) + 1 + \bar{\theta}\log_2(e)(t_k - t_0) + \log_2(\epsilon(t_0))\right].$$

- In the general case, only an implicit characterization
- Effect of non-instant communication (through T_M) has only a "transient" effect on sufficient data rate
- In the scalar case, if no disturbance then necessary and sufficient asymptotic data rates are same

Shared communication resource



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- Channel may not be available during some intervals (blackouts)
- Time-triggered strategies would be very conservative
- Event-triggered controllers typically assume *on-demand* availability of channel

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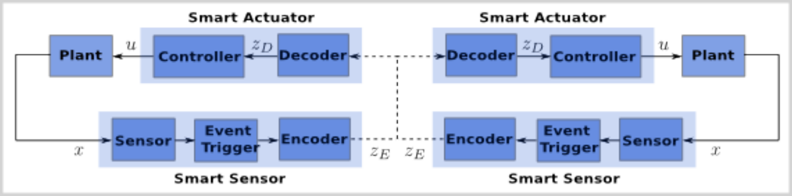

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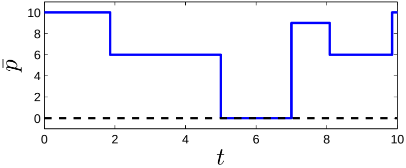
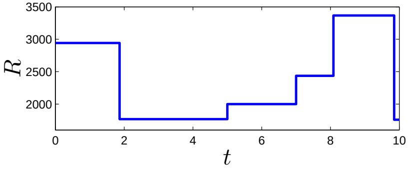
Key to online state based transmission policy: data capacity

Time-slotted channel model





$$R(t) = R_j, \quad \forall t \in (\theta_j, \theta_{j+1}], \quad \textbf{min comm. rate: } \frac{p_k}{\Delta(t_k, p_k)} \geq R(t_k)$$

$$\bar{p}(t) = \bar{\pi}_j, \quad \forall t \in (\theta_j, \theta_{j+1}], \quad \textbf{max packet size: } p_k \leq \bar{p}(t_k)$$

- j^{th} time-slot is of length $T_j = \theta_{j+1} - \theta_j$
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- Channel evolution is known a priori

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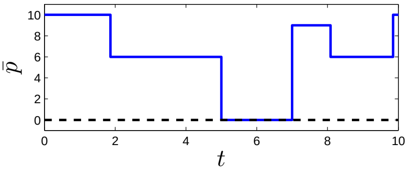

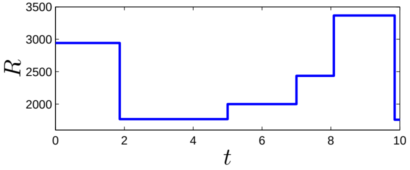

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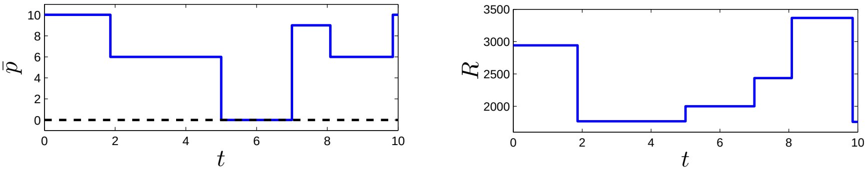

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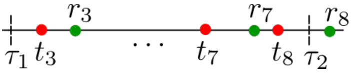
Need to quantify *data capacity*

Data capacity

max # of bits that can be *communicated* during the time interval $[\tau_1, \tau_2]$, overall all possible $\{t_k\}$ and $\{p_k\}$

$$\mathcal{D}(\tau_1, \tau_2) \triangleq \max_{\substack{\{t_k\},\{p_k\} \\ \text{s.t. } \ldots}} n \sum_{k=\underline{k}_{\tau_1}}^{\overline{k}_{\tau_2}} p_k$$



$\underline{k}_{\tau_1} = 3$, $\overline{k}_{\tau_2} = 7$

Data capacity

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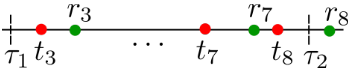


$\underline{k}_{\tau_1} = 3$, $\overline{k}_{\tau_2} = 7$

Equivalent to optimal allocation of *discrete* # bits to be transmitted in each time slot

Data capacity as allocation problem

Max # bits that may be transmitted in slot j

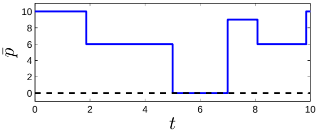
$$n\phi_j \le \begin{cases} nR_jT_j + n\bar{\pi}_j, & \text{if } \bar{\pi}_j > 0 \\ 0, & \text{if } \bar{\pi}_j = 0 \end{cases}$$

Available time in slot j is affected by prior transmissions

$$n\phi_j \le \begin{cases} nR_j\bar{T}_j(\phi_{j_0}^{j_f}) + n\bar{\pi}_j, & \text{if } \bar{T}_j(\phi_{j_0}^{j_f}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

Count only the bits also received

$$\frac{\phi_j}{R_j} \le \begin{cases} \bar{T}_j(\phi_{j_0}^{j_f}) + \theta_{j_f} - \theta_{j+1}, & \text{if } \bar{T}_j(\phi_{j_0}^{j_f}) > 0 \\ 0, & \text{otherwise.} \end{cases}$$



$$\mathcal{D}(\theta_{j_0}, \theta_{j_f}) = \max_{\substack{\phi_j \in \mathbb{Z}_{\ge 0} \\ \text{s.t. } \dots}} n \sum_{j=j_0}^{j_f - 1} \phi_j .$$

A suboptimal solution for "slowly varying channels"

Proposition

Assume $\dfrac{\bar{\pi}_j}{R_j} < T_{j+1}, \forall j \in \mathcal{N}_{j_0}^{j_f}$ (*any bits transmitted in slot* j *are received before the end of slot* $j+1$).

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Assume $\frac{\bar{\pi}_j}{R_j} < T_{j+1}, \forall j \in \mathcal{N}_{j_0}^{j_f}$ *(any bits transmitted in slot j are received before the end of slot $j+1$). Let* $\phi^r = \underset{\substack{\phi_j \in \mathbb{R}_{\geq 0} \\ s.t. \ \ldots}}{\operatorname{argmax}} \sum_{j=j_0}^{j_f-1} \phi_j$ *(LP).*

Let

$$\phi^N \triangleq \lfloor \phi^r \rfloor \triangleq (\lfloor \phi^r_{j_0} \rfloor, \ldots, \lfloor \phi^r_{j_f-1} \rfloor), \quad \mathcal{D}_s(\theta_{j_0}, \theta_{j_f}) \triangleq n \sum_{j=j_0}^{j_f-1} \phi_j^N.$$

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Then

- ϕ^N *is a sub-optimal solution*
- $\mathcal{D}(\theta_{j_0}, \theta_{j_f}) - \mathcal{D}_s(\theta_{j_0}, \theta_{j_f}) \leq n(j_f - 1 - j_0)$.

Real time computation of data capacity

Proposition

Let ϕ^* (or ϕ^N) be any optimizing solution to $\mathcal{D}(\theta_{j_0}, \theta_{j_f})$ (or $\mathcal{D}_s(\theta_{j_0}, \theta_{j_f})$).

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$$\hat{\mathcal{D}}(t, \theta_{j_f}) \triangleq \left[n \left\lfloor \phi^*_{j_0} - R_{j_0}(t - \theta_{j_0}) \right\rfloor\right]_+ + n \sum_{j=j_0+1}^{j_f-1} \phi^*_j$$

$$\hat{\mathcal{D}}_s(t, \theta_{j_f}) \triangleq \left[n \left\lfloor \phi^N_{j_0} - R_{j_0}(t - \theta_{j_0}) \right\rfloor\right]_+ + n \sum_{j=j_0+1}^{j_f-1} \phi^N_j,$$

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Significance: Sufficient to solve the data capacity problem for intervals $[\theta_{j_0}, \theta_{j_f}]$ of interest.

Elements of the event-trigger

Recall performance objective: ensure $h_{\mathrm{pf}}(t) \triangleq \frac{V(x(t))}{V_d(t)} \leq 1$, for all $t \geq t_0$

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If $h_{\mathrm{pf}}(t) \leq 1$ *and* $h_{\mathrm{ch}}(t) \leq 1$ *then* $h_{\mathrm{pf}}(s) \leq 1, \forall s \in [t, t + T']$.

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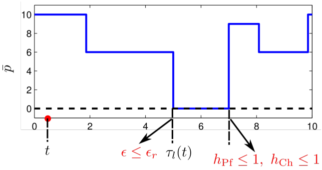
$$\tilde{\mathcal{L}}_1(t) \triangleq \bar{h}_{\mathrm{pf}}\left(\mathcal{T}(t), h_{\mathrm{pf}}(t), \epsilon(t)\right)$$

$$\tilde{\mathcal{L}}_2(t) \triangleq \bar{h}_{\mathrm{ch}}\left(\mathcal{T}(t), h_{\mathrm{pf}}(t), \epsilon(t), \psi^{\tau_l}(t)\right)$$

$$\mathcal{T}(t) \triangleq \begin{cases} T_M(\psi^{\tau_l}(t)), & \text{if } \psi^{\tau_l}(t) \geq 1 \\ \frac{2}{R(t)}, & \text{if } \psi^{\tau_l}(t) = 0. \end{cases}$$

Role of data capacity in control



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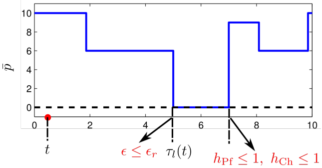

$$\tilde{\mathcal{L}}_3(t) \triangleq n \log_2 \left(\frac{e^{\bar{\mu}(\tau_l(t)-t)}\epsilon(t)}{\epsilon_r(t)} \right) - \sigma_1 \hat{\mathcal{D}}_s(t, \tau_l(t))$$

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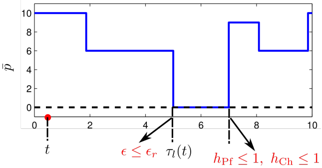

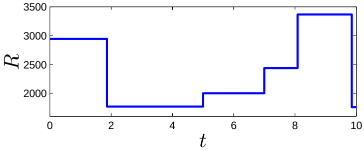

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Transmission policy should be in tune with the optimal allocation

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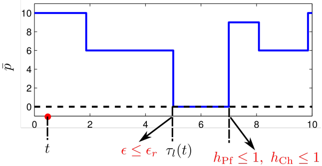

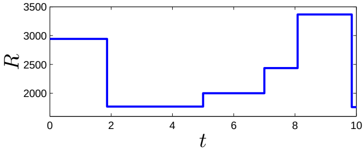

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$$\Phi^{\tau_l}(t) \triangleq \left[\lfloor \mathcal{P}_j - R_j(t-\theta_j) \rfloor \right]_+ ,\ t \in (\theta_j, \theta_{j+1}] \text{ (optim. alloc. in } (t, \theta_{j+1}])$$

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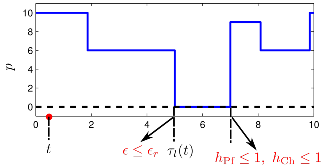

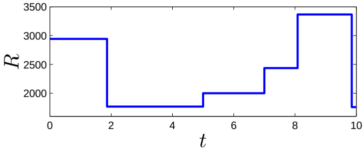

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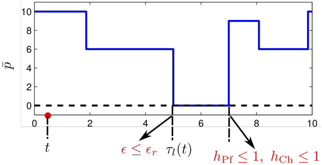

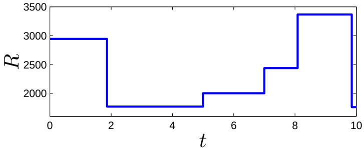

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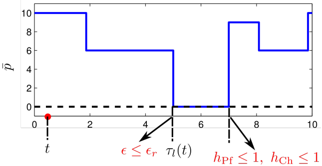

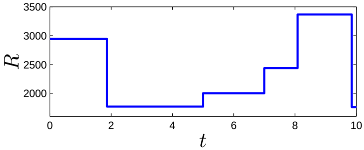

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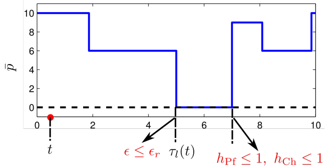
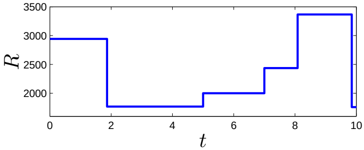
Role of data capacity in control





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But $\psi^{\tau_l}(t)$ can be 0 when $\bar{p}(t) > 0$ (artificial blackouts)

Control policy in the presence of blackouts

$$t_{k+1} = \min\Big\{t \geq \tilde{r}_k : \psi^{\tau_l}(t) \geq 1 \wedge \Big(\max\{\tilde{\mathcal{L}}_1(t), \tilde{\mathcal{L}}_1(t^+), \tilde{\mathcal{L}}_2(t), \tilde{\mathcal{L}}_2(t^+)\} \geq 1 \vee \max\{\tilde{\mathcal{L}}_3(t), \tilde{\mathcal{L}}_3(t^+)\} \geq 0\Big)\Big\},$$

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\end{aligned}
$$

$$
\begin{aligned}
& p_k \in \mathbb{Z}_{>0} \cap [\underline{p_k}, \psi^{\tau_l}(t_k)] \\
& \underline{p_k} \triangleq \min\{p \in \mathbb{Z}_{>0} : \bar{h}_{\text{ch}} \left(T_M(p), h_{\text{pf}}(t_k), \epsilon(t_k), p\right) \leq 1\}.
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$$\tilde{r}_k = \min\{t \geq r_k : \psi^{\tau_l}(t) \geq 1 \ \vee \ \bar{p}(t) = 0\}.$$

Control policy in the presence of blackouts

Theorem

If

- $R(t) \geq \frac{(p+2)}{T_M(p)},\ \forall p \in \{1, \ldots, p^{Max}\},\ \forall t$
- $\tilde{\mathcal{L}}_1(t_0) \leq 1,\ \tilde{\mathcal{L}}_2(t_0) \leq 1$ *and* $\tilde{\mathcal{L}}_3(t_0) \leq 0$ *(initial feasibility)*
- *Conditions on blackout lengths*

Control policy in the presence of blackouts

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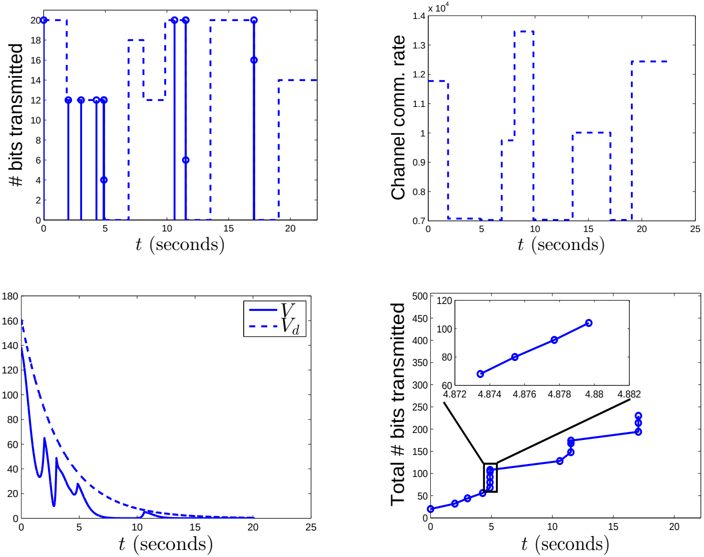
- *Conditions on blackout lengths*

Then

- $\{t_k\}, \ \{p_k\}, \ \{\tilde{r}_k\}$ *well defined*
- *inter-transmission times have uniform positive lower bound*
- $V(x(t)) \leq V_d(t_0)e^{-\beta(t-t_0)}$ *for* $t \geq t_0$ *(origin is exponentially stable)*

Simulation results: 2D linear system



Outline

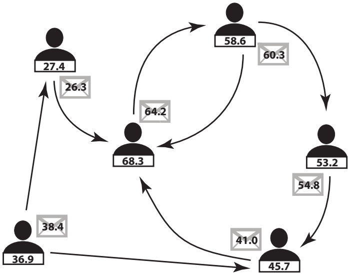
Privacy and security in CPS

- Malicious attacks can have catastrophic physical consequences - industrial plants, cars and traffic, medical devices

Privacy and security in CPS

- Malicious attacks can have catastrophic physical consequences - industrial plants, cars and traffic, medical devices
- Large scale collection of user data in many domains - many benefits but loss of individuals' privacy
- Encryption not sufficient - need a multi-layered approach

Differential privacy



Definition (Differential privacy)

Given $\delta, \epsilon \in \mathbb{R}^n_{\geq 0}$, the mechanism $\mathcal{M}$ is ϵ-differentially private if, for any two δ-adjacent data $X^{(1)}$ and $X^{(2)}$ and any observation set $\mathcal{O}$, one has

$$\mathbb{P}\{\mathcal{M}(X^{(2)}) \in \mathcal{O}\} \leq e^{\epsilon}\mathbb{P}\{\mathcal{M}(X^{(1)}) \in \mathcal{O}\}$$

Differentially private average consensus

Agents' dynamics: $\theta(k+1) = \theta(k) - hLx(k) + S\eta(k), \quad \theta \in \mathbb{R}^n$

Messages: $x(k) = \theta(k) + \eta(k)$

h is step size, S is a diagonal matrix with diagonal $(s_1, \ldots, s_n)$

$\eta_i(k) \in \mathbb{R}$ is the noise added by agent i on time step k

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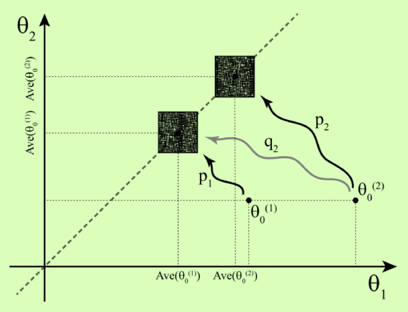
Objective:

- Design the distribution of the noise sequences η
- Want asymptotic average consensus and ϵ-differential privacy of the initial condition,
- ϵ as small as possible, and maximize algorithms accuracy

An impossibility result

Theorem

For any $\delta, \epsilon > 0$, agents cannot simultaneously converge to the average of their initial states in distribution and preserve ϵ-differential privacy of their initial states.



Contradiction: $p_1, p_2 \simeq 1, q_2 \simeq 0, p_1 \leq e^{\epsilon} q_2$

Differentially private average consensus

Theorem

If

- $\eta_i(k) \sim Lap(b_i(k))$ *(Laplace distribution)*
  $b_i(k) = c_i q_i^k, \ c_i \in \mathbb{R}_{>0}, \ q_i \in (|s_i - 1|, 1), \ s_i \in (0, 2)$

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Then

- *For all* $i \in \{1, \ldots, n\}$, $\theta_i(k) \to \theta_\infty$ *almost surely, where*
  $\theta_\infty = \mathrm{Ave}(\theta(0)) + \sum_{i=1}^n \frac{s_i}{n} \sum_{j=0}^\infty \eta_i(j)$
- $\mathbb{E}\{\theta_\infty\} = \mathrm{Ave}(\theta(0)), \quad var\{\theta_\infty\} = \frac{2}{n^2} \sum_{i=1}^n \frac{s_i^2 c_i^2}{1-q_i^2}$
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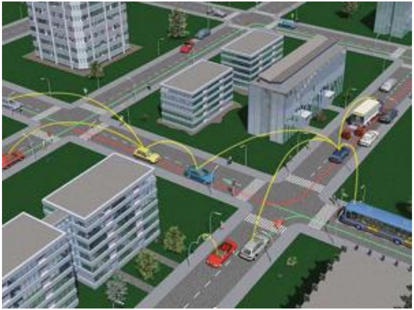
Optimal selection of noise parameters by minimizing var $\{\theta_\infty\}$

Outline

Networked transportation systems

- Collision avoidance, cruise control, trip planning, traffic coordination, on-demand public transport, multi-modal coordination . . .

Intersection traffic coordination



Source: CAR 2 CAR communication consortium

- Vehicle-to-vehicle and vehicle-to-infrastructure communication can be used to coordinate traffic - no traffic lights
- Individual vehicles can use fore-knowledge of the schedule to optimize their travel much before they reach the intersection
- Potential to significantly improve safety, travel ease, travel times, energy consumption

Problem statement

- Assumptions: (i) Single lane in each direction, (ii) all vehicles are identical with length L, (iii) no turning at the intersection, (iv) no sources or sinks for vehicles along the branches.

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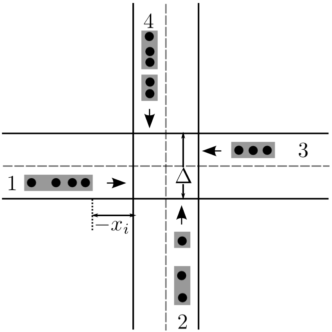
- Objective: Design a traffic coordination mechanism for networked and automated vehicles that seeks to minimize the cost function

- Challenges: Problem is combinatorial. Solving it at the level of individual cars is computationally expensive and not scalable.

A scalable solution



- Black dots are individual vehicles
- Vehicles are clustered into *bubbles* represented by the grey boxes
- Vehicles of a bubble *platoon (rigid cohesive group)* when crossing the intersection
- x_i is the position of the lead vehicle in the bubble
- Δ is the length of the intersection

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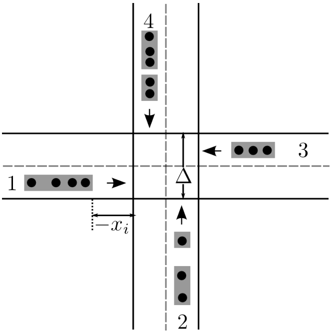

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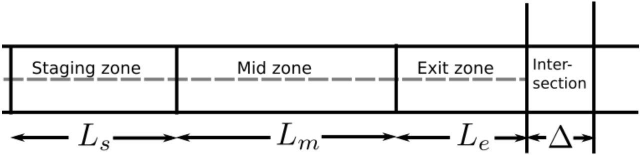
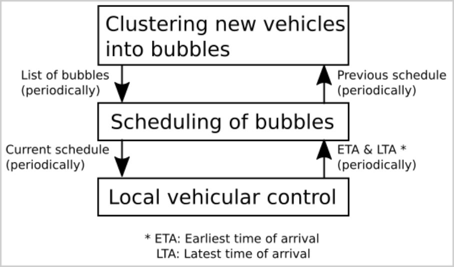


Overview of hierarchical solution



State of bubble i:

$(x_i, v_i, m_i, \bar{\tau}_i^{\text{occ}}, \mathcal{I}_i) \in \mathbb{R}^4 \times \{1, 2, 3, 4\}$,

x_i: position of the lead vehicle

v_i: velocity of the lead vehicle

m_i: number of vehicles in the bubble

$\bar{\tau}_i^{\text{occ}}$: guaranteed upper-bound on τ_i^{occ}

$\mathcal{I}_i$: branch label that the bubble is on

τ_i: scheduled time of approach at the beginning of the intersection for the lead vehicle in bubble i

τ_i^{occ}: occupancy time - time for which bubble i occupies the intersection

Scheduling of bubbles

Constraints:

$$\tau_i \in [\max\{\tau^{\min}, \tau_i^e\}, \tau_i^l], \quad \text{interval determined by initial conditions}$$
$$\tau_j \geq \tau_i + \bar{\tau}_i^{\mathrm{occ}}, \quad \text{if bubbles } i \text{ and } j \text{ on same branch and } j \text{ follows } i$$
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Simplified cost function for scheduling:

$$\mathcal{C}_{\mathcal{L}} \triangleq \sum_{i \in \mathcal{L}} m_i (W_T(\tau_i - t_s) + F_i(\bar{v}_i)) = \sum_{i \in \mathcal{L}} m_i \Big(W_T \frac{-x_i}{\bar{v}_i} + F_i(\bar{v}_i) \Big)$$

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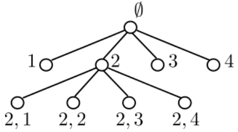
Schedule optimization using *branch and bound*.



Tree of possible bubble passage orders.

Safe-following distance

Definition (Safe-following distance)

We say a quantity $\mathcal{D}(v^v_{j-1}(t), v^v_j(t))$ is *a safe-following distance* at time t for the pair of vehicles $j-1$ and j if

- $x^v_{j-1}(t) - x^v_j(t) \geq \mathcal{D}(v^v_{j-1}(t), v^v_j(t))$
- both the vehicles were to perform the maximum braking maneuver

then the two vehicles would be safely separated, $x^v_{j-1} - L \geq x^v_j$ until they come to a complete stop.

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Lemma

$\mathcal{D}(v^v_{j-1}(t), v^v_j(t)) = L + \max\left\{0, \frac{-1}{2u_m}\left((v^v_j(t))^2 - (v^v_{j-1}(t))^2\right)\right\}$ *is a safe-following distance for a vehicle j following $j-1$.*

Safety ratio: $\sigma_j(t) \triangleq \dfrac{x^v_{j-1}(t) - x^v_j(t)}{\mathcal{D}(v^v_{j-1}(t), v^v_j(t))}$

Distributed vehicular control

Consists of two parts

- an *uncoupled optimal feedback controller* for reaching the intersection at a nominal deadline with a nominal speed: g_{uc}
- a *controller for safe following*: $g_{sf} \triangleq \min\{g_{uc}, g_{us}\}$,

$$g_{us}(\zeta_j, u^v_{j-1}) \triangleq \begin{cases} u^v_{j-1}, & \text{if } v^v_j = 0, \\ \left(\frac{v^v_{j-1}}{v^v_j}\left(1 + \sigma_j \frac{u^v_{j-1}}{-u_m}\right) - 1\right)\left(\frac{-u_m}{\sigma_j}\right), & \text{if } v^v_j > 0. \end{cases}$$

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Control law: $$u^v_j(t) = \begin{cases} g_{uc}, & \text{if } \zeta_j \notin \mathcal{C}_s,\ v^v_j < v^M, \\ [g_{uc}]^0_{u_m}, & \text{if } \zeta_j \notin \mathcal{C}_s,\ v^v_j = v^M, \\ g_{sf}, & \text{if } \zeta_j \in \mathcal{C}_s,\ v^v_j < v^M, \\ [g_{sf}]^0_{u_m}, & \text{if } \zeta_j \in \mathcal{C}_s,\ v^v_j = v^M. \end{cases}$$

Coupling set: $\mathcal{C}_s \triangleq \{(\mathrm{v}_1, \mathrm{v}_2, \sigma) : \mathrm{v}_2 \geq \mathrm{v}_1 \text{ and } \sigma \in [1, \sigma_0]\}$

Provably safe traffic coordination

Theorem

If

- *Exit zone length* $L_e \geq -\frac{(v^M)^2}{2u_m} + \frac{(\nu^{nom})^2}{2u_M}$
- *New vehicles arrive at a safe following distance*

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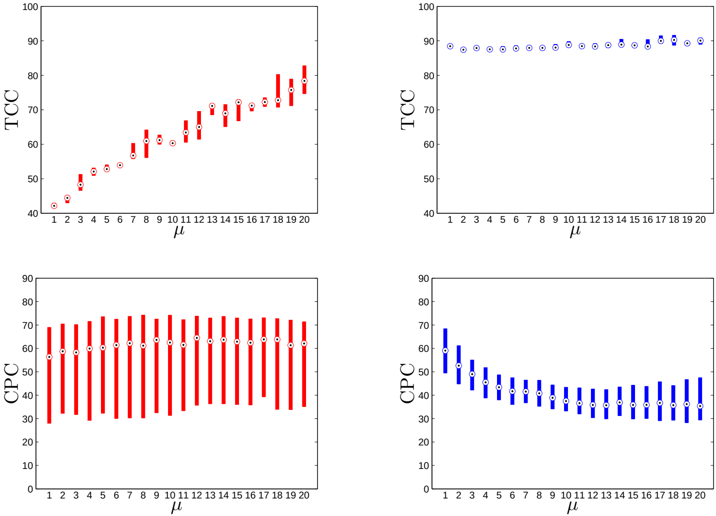
- *New vehicles arrive at a safe following distance*

Then

- *Each vehicle belongs to some bubble*
- *Each bubble scheduled at least once*
- *Feasible schedule always exists*
- *Inter-vehicle safety is ensured for all vehicles at all times*
- *Distributed vehicular control respects the prescribed occupancy schedule*

Videos

Simulations



Outline

Opportunistic state-triggered control

- Fusion of event-triggered control and information-theoretic control
- Control under bounded and specified channel capacity
- Stabilization with prescribed convergence rate
- Analysis of average data rate
- Control under time-varying channels (including blackouts)

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Future plans:

- Fusion of event-triggered control and information-theoretic control for nonlinear systems and distributed control
- Stochastic channel models
- More realistic scheduling constraints

- Open problem: analytical quantification of the average data rate for an arbitrary event based controller

Differential privacy in CPS

- Differentially private average consensus
- Fundamental trade-off between accuracy and privacy
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Future plans:

- Fundamental data rate theorems under privacy requirements

- State-triggered control works by encoding the control goal in the event-trigger and the aperiodic transmission instants carry information - what are the implications for privacy?

Networked transportation systems

- A scalable hierarchical-distributed solution to coordination of intersection traffic applicable to a wide range of traffic densities
- A provably safe online coordination of traffic
- Framework has the potential to significantly improve safety, travel ease, travel time and energy consumption

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Future plans:

- Incorporate statistical and real-time data of incoming traffic
- Extend to a network of intersections
- Multiple temporal and spatial refinements of data and control

- Privacy, security and resilience
- On-demand routing and scheduling of bus services

- Experiments and implementation in lab

Teaching plans

- UG courses: Control systems, signal processing, linear systems, linear algebra, circuit theory and dynamics
- PG courses: Linear systems theory, random processes, nonlinear systems, hybrid systems, distributed control, networked control systems
- Course on CPS & CPS applications - possibly collaborate with other departments, encourage students to do multi-disciplinary, multi-domain projects

Acknowledgements

Funding from National Science Foundation