Speed-Boosted Adaptation and Applications to Swarms and Clusters of High-Performance Aerospace Systems

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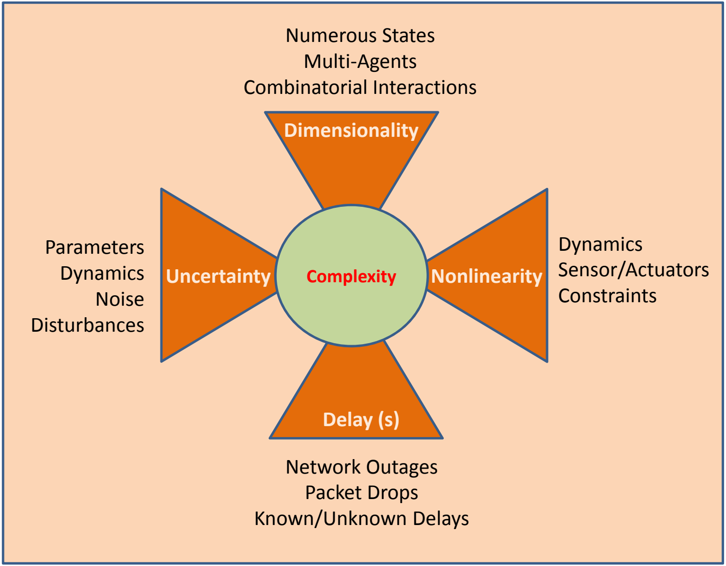
Systems & Control Engineering Seminar
Indian Institute of Technology, Bombay

Control of Agile Responsive Aerospace Systems



Nonlinear Stability and Control Design

Aleksandyr
Mikhailovich
Lyapunov

- Lyapunov-like techniques are usually the basis of most nonlinear stability analysis
- Controllers are synthesized to suit "chosen" Lyapunov functions
- Lyapunov's Direct Method paired with LaSalle invariance, Barbalat's lemma establish foundations for optimal, robust, and adaptive control
- Finding the right Lyapunov function is more art than science

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Consider a really simple stabilization problem

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= W(x)\theta^* + u \end{aligned} \qquad \left\{ \begin{array}{r} u = -k_p x_1 - k_v x_2 - W(x)\theta^* \\ k_p > 0,\ k_v > 0 \end{array} \right.$$

The closed-loop system is UES. This can be established via

$$A_m \doteq \begin{bmatrix} 0 & 1 \\ -k_p & -k_v \end{bmatrix}, \quad A_m^T P + P A_m = -Q, \quad V_1 = \mathbf{x}^T P \mathbf{x} \rightarrow \dot{V}_1 = -\mathbf{x}^T Q \mathbf{x} < 0$$

Alternately, we could consider "energy-like" function

$$V_2 = (k_p x_1^2 + x_2^2)/2 \quad \rightarrow \quad \dot{V}_2 = -k_v x_2^2 \leq 0$$

Thus, $V_2(\mathbf{x})$ is "non-strict" (aka *defective*) but the story still has a happy ending, thanks to LaSalle Invariance, Barbalat's Lemma...

Questions: How do we construct strict Lyapunov functions? Why bother about them?

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Constructing Strict Lyapunov Functions

- If UGAS is already known, converse theory guarantees existence
- Explicit availability of a strict Lyapunov function aids robustness analysis (external disturbances, adaptive control, time-delays, . . .)
- Construction is a challenging problem, significant ongoing research (Mazenc, Malisoff, Teel, Nesic, etc.)

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- Higher-order Lie derivatives of non-strict Lyapunov functions
- Use of continuous-time Matrosov theorem
- Feedback with small gains
- Sufficient conditions, usually non-quadratic functions

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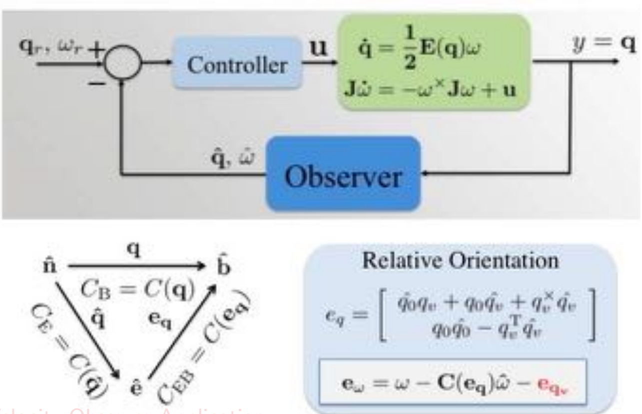
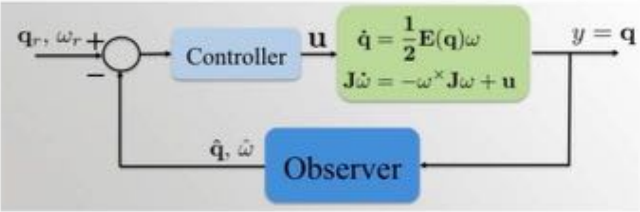
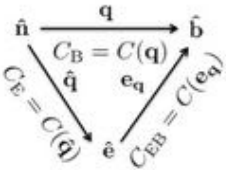


Angular Velocity Observer Application:

- Salcudian 1991, Open Problem (till Chunodkar, Akella, JGCD 2014)
- Switching provides *strictification* while ensuring C^0 continuity of states
- Finite number of switches - no zeno-type behavior
- Smooth analog for this result available through a spiral design approach (Thakur, Mazenc, Akella, JGCD 2015)

Strictification via State-Dependent Switching





Relative Orientation

$$e_q = \begin{bmatrix} \hat{q}_0 q_v + q_0 \hat{q}_v + q_v^{\times} \hat{q}_v \\ q_0 \hat{q}_0 - q_v^{\mathrm{T}} \hat{q}_v \end{bmatrix}$$

$$\boxed{\mathbf{e}_\omega = \omega - \mathbf{C}(\mathbf{e_q})\hat{\omega} - \mathbf{e}_{\mathbf{q}_v}}$$

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Strictification via Filter Embedment

Introduce stable linear low-pass filters (i.e., $\alpha > 0$)

$$\dot{x}_{1f} = -\alpha x_{1f} + x_1, \qquad \dot{x}_{2f} = -\alpha x_{2f} + x_2$$
$$\dot{u}_f = -\alpha u_f + u, \qquad \dot{W}_f = -\alpha W_f + W(\boldsymbol{x})$$

Simple algebra results in the following, modulo exponentially decaying terms,

$$\begin{aligned} \dot{x}_{1f} &= x_{2f} \\ \dot{x}_{2f} &= W_f\theta^* + u_f \end{aligned} \qquad \begin{cases} u_f = -k_p x_1 - k_v x_{2f} - W_f\theta^* \\ k_p > 0,\ k_v > 0 \end{cases}$$

Now, consider the Lyapunov function candidate $V_3(\boldsymbol{x}) = (x_1^2 + x_{2f}^2)/2$,

$$\dot{V}_3 = -k_p x_1^2 - k_v x_{2f}^2 + (\alpha - k_p - k_v)x_1 x_{2f}$$

Selecting the filter gain $\alpha = (k_p + k_v)$ results in $\dot{V}_3 = -k_p x_1^2 - k_v x_{2f}^2$

Mr. Lyapunov is both QUADRATIC and STRICT again!

The control signal can be recovered by

$$u = \dot{u}_f + \alpha u_f \quad \Longrightarrow \quad u = -\alpha k_p x_1 - (k_p + k_v)x_2 - W(\boldsymbol{x})\theta^*$$

Thus, for this academic example, filters are for analysis ONLY and they aren't needed for implementation!

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Strictification - Why Bother?

Suppose θ^* is unknown, constant and we consider adaptation

$$u = -k_p x_1 - k_v x_2 - W(\boldsymbol{x})\hat{\theta}(t)$$

The update law for the parameter estimate $\hat{\theta}(t)$ can be established through

$$V = \frac{k_p}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta}, \quad \tilde{\theta} \doteq \hat{\theta} - \theta^*, \quad \gamma > 0$$

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Big Trouble! We are staring at the Uniform Detectability Obstacle

Fix: Either introduce non-intuitive cross-terms to "strictify" the Lyapunov function or, possibly adopt the filter embedment approach

And.. this is only the tip of the iceberg..

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- Mature subject area
- Several variants exist
  - Direct/Indirect
  - Backstepping
  - Immersion & Invariance
  - $\mathcal{L}_1$ Adaptive Control

Procustes' *Mythical* Bed

- **Fact 1:** Even a linear plant under the action of an adaptive controller becomes nonlinear in the closed-loop due to the adaptation mechanism
- **Fact 2:** Plant parameters affine in the governing dynamic model
- **Fact 3:** Parameter estimates converge to their true values only under suitable persistence excitation (PE) conditions
- **Fact 4:** Most existing designs based on the Certainty Equivalence (CE) Principle

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Certainty Equivalence Principle

- Consider a prototypical adaptive stabilization problem
  - Suppose all plant parameters $\boldsymbol{\theta}^*$ are known and

$$\boldsymbol{u} = \mathbf{k}(t)h(\boldsymbol{x}) + W(\boldsymbol{x})\boldsymbol{\theta}^*$$

is the controller that achieves the desired control objective

  - Then, in the case $\boldsymbol{\theta}^*$ is unknown, design controller

$$\boldsymbol{u} = \mathbf{k}(t)h(\boldsymbol{x}) + W(\boldsymbol{x})\hat{\boldsymbol{\theta}}(t)$$

together with a suitable update law of $\hat{\boldsymbol{\theta}}(t)$ (parameter estimator) so that the closed-loop is stable and the control objective is again achieved.

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Deviating from the CE Formalism

- Add tuning function $\beta(x)$ to the adaptation (Astolfi & Ortega, IEEE TAC, 2003) :

$$\boldsymbol{u} = \mathbf{k}(t)h(\boldsymbol{x}) + W(\boldsymbol{x})\left[\hat{\boldsymbol{\theta}}(t) + \boldsymbol{\beta}(\boldsymbol{x})\right]$$

- The state-dependent tuning function $\beta(\boldsymbol{x})$ should satisfy an integrability condition:

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  - Sufficient condition ONLY (... think $A^T P + PA = -Q$)
  - $Q(x)$ is a design function
  - β is not uniquely defined
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- Nonlinear single-input systems in cascade form and linear multi-input systems always satisfy the manifold attractivity condition (Akella, Subbarao, SCL 2005)
- Stability analysis:

$$\begin{aligned} V &= \frac{1}{2}\boldsymbol{x}^T\boldsymbol{x} + \frac{\sigma}{2}\boldsymbol{z}^T\boldsymbol{z}, \qquad \boldsymbol{z} \doteq \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^* + \boldsymbol{\beta}, \quad \sigma > 0, \\ \dot{V} &= -\boldsymbol{x}^T\boldsymbol{x} - \boldsymbol{x}^T W(\boldsymbol{x})\boldsymbol{z} - \sigma\|W(\boldsymbol{x})\boldsymbol{z}\|^2 \quad \leq 0 \end{aligned}$$

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Comparing the Two Designs

- CE based designs typically result in

$$\begin{aligned}\dot{\boldsymbol{x}} &= -\boldsymbol{x} - W(\boldsymbol{x})\tilde{\boldsymbol{\theta}}; \quad \{\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^* \\ \dot{\hat{\boldsymbol{\theta}}} &= \gamma_{ce} W^T(\boldsymbol{x})\boldsymbol{x}\end{aligned}$$

  - Performance ultimately dictated by parameter estimator $\Rightarrow W(\boldsymbol{x})\tilde{\boldsymbol{\theta}}$ like disturbance
  - Parameter estimates driven by the regulating/tracking error
  - Unable to mimic $\dot{\boldsymbol{x}} = -\boldsymbol{x}$

- The attracting manifold design, on the other hand, results in

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Speed-Boosted Adaptation

- Wash all states and the regressor through stable linear low-pass filters
- The regressor filter assures circumvention of the integrability obstacle. Specifically, $\beta = W_f^T \mathbf{x}_f$ satisfies the integrability condition
- The closed-loop system becomes

$$\begin{aligned} \dot{\mathbf{x}}_f &= -\mathbf{x}_f - W_f(t)\mathbf{z} \\ \dot{\mathbf{z}} &= -\gamma W_f^T W_f \mathbf{z} \end{aligned}$$

- Very high-dimensional closed-loop system ($x_f \in \mathcal{R}^n$; $W_f \in \mathcal{R}^{n\times p}$)
- Speed boosting: $\mathbf{k}(t) = k\rho(t)r^2(t) \quad k > 0, \inf_{t\geq 0}\rho(t) = \rho^* > 0$
  - Scalar extension: (non-filter, $\rho(t) = 1$); $\mathbf{k} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ (Yang, Akella, SES 2015)
  - Second-order dynamic extension (Filter)
    $\dot{\rho} = -(\rho - 1/r^2 - \varepsilon), \quad \rho(0) = 1/r^2(0) + \varepsilon, 0 < \varepsilon \ll 1$
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Adaptive Attitude Tracking Application

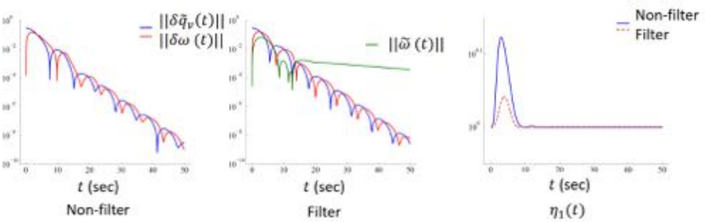
- System parameters and reference rate profile

(Seo, Akella, JGCD 2008)

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix}$$

$$\omega_r(t) = 0.3(1 - e^{-0.01t^2})\cos t + te^{-0.01t^2}(0.08\pi + 0.006 \sin t)$$

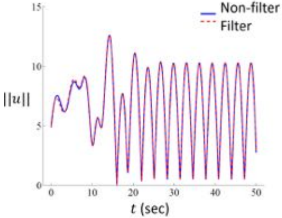
- Tracking errors and dynamic gain $\mathbf{k}(t) = \eta_1(t)$

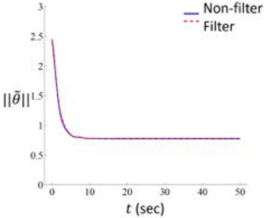


(Yang, Akella, AIAA/AAS SFM, 2016)

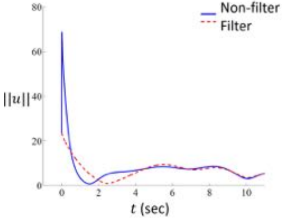
Simulation Results

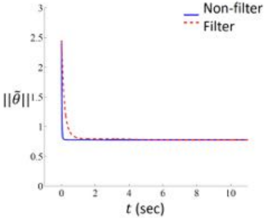
- Control and estimation error norms





- Large initial rate error: $\|\delta\boldsymbol{\omega}(0)\| = \sqrt{3}$

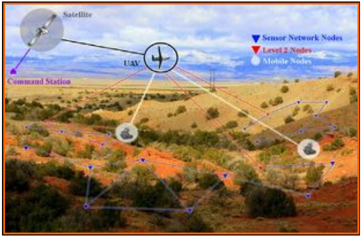




Coordinated Sensing and Decentralized Control

Distributed Heterogeneous Networks:

- Mission and task decomposition
- Minimal communication, persistence
- Coordination, optimality and constraint satisfaction

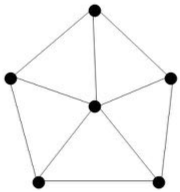


Research Focus:

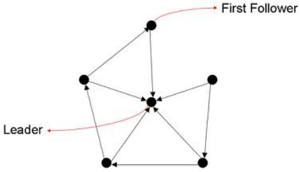
- Self organization – clustering
- GPS-denied navigation, path-planning
- Consensus establishment
- Time-delay in communication

Swarms and Information Architectures

- Undirected/Symmetric
- Rigid, but not minimally so



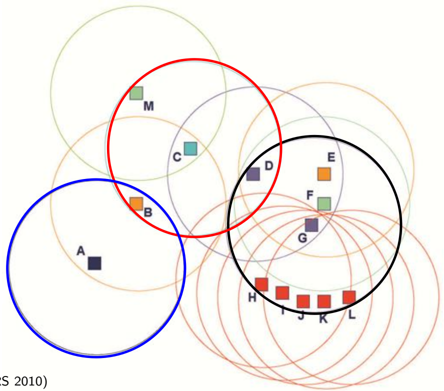
- Directed/Asymmetric
- Minimally persistent
- LFF, LRF, Co-Leader



Clustering for Self Organization

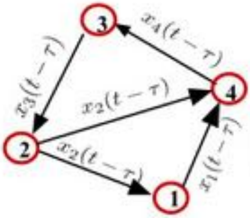
Hierarchial Self-Organization of the Network

- Determination of clusterheads and clients
- Optimization a very difficult problem (NP hard)
- Best approximations $\sim O(\log n)$ for 1-D; $O(\sqrt{n})$ for 2-D
- Mobile nodes not involved



(Mercker, Akella, Alvarez, JIRS 2010)

Time-Delays & Imperfect Communication



$$\ddot{x}_i(t) = \alpha \sum_{j \in N_i} \bar{a}_{ij}(x_j(t-\tau) - x_i(t)) - \beta \dot{x}_i(t)$$

No self-delay protocol

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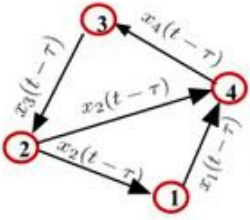
Same self-delay protocol

Dynamics with Unstable Drift:

- Graph containing *spanning tree* necessary for consensus
- Necessary and sufficient stability conditions for *cyclic graphs* in terms of control gains α (position-feedback) and β (rate-feedback)
- Directed graphs are *less robust* w.r.t. time-delay uncertainty when compared to corresponding underlying undirected graphs



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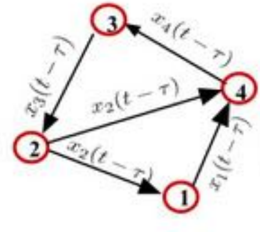

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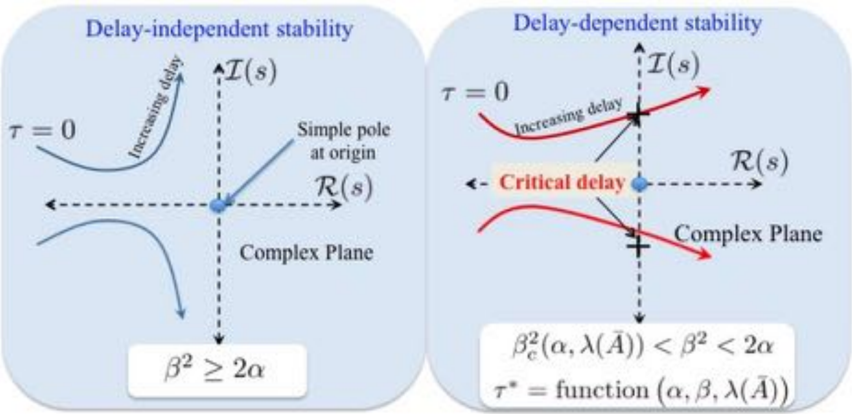
Stability Conditions - No Self-Delay Protocol

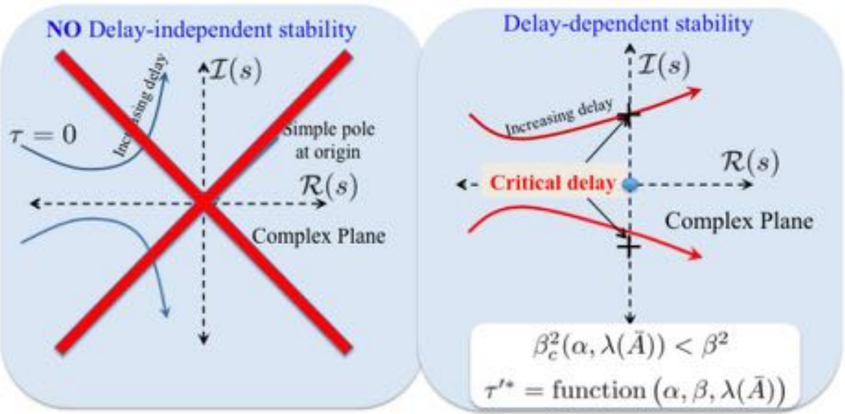
- No self-delay, weighted adjacency matrix $\bar{A}$
- Critical delay $\tau^* \leq \tau_{\max}$

(Yang, Mazenc, Akella, JGCD 2015)



Stability Conditions - with Same Self-Delay

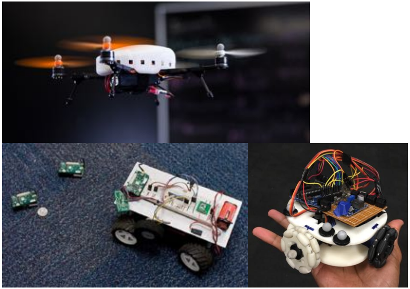
- Same self-delay, weighted adjacency matrix $\bar{A}$
- Critical delay $\tau^* \leq \tau_{\max}$



Autonomous Guidance for Satellite Swarms

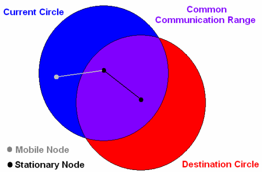
Swarm about Dwarf Planet Ceres
(Hernandez, Thakur, Akella, JGCD 2015)

Navigation without Localization/GPS Infrastructure



GPS Denied Robot Navigation:

- Reach “purple” from “blue”
- Arbitrary heading
- Imperfect communication boundaries



Vision-Enabled Autonomy: Pose and Tumble Estimation

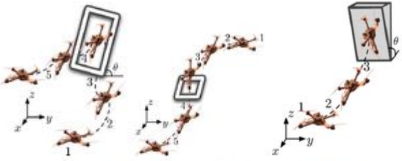
Vision-Based Discrete Adaptive Rate Estimation
(Almeida, Akella, Mortari, AIAA/AAS SFM 2016)

What if Landmarks aren't Mapped?

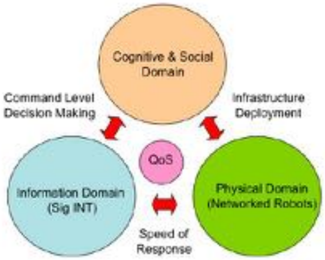
Simultaneous Localization and Mapping (15 fps)
(OrbSLAM; Mur-Artal et al. Universidad Zaragoza 2016)

Challenges, Opportunities, Future Work..

- Distributed, ubiquitous
- "Internet of Things" – at massive scales
- Human-robot interface, perception, cognition



(a) Vertical opening (b) Horizontal opening (c) Perching



- Layered-autonomy, dependability
- Uncertainty, quantification and its impact on sensor/resource allocation

Challenges, Opportunities, Future Work..

- Distributed, ubiquitous
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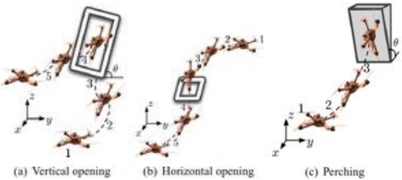

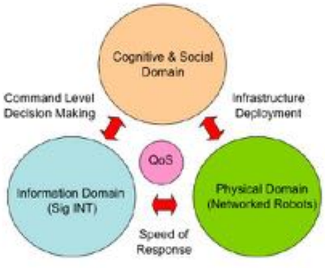

- Layered-autonomy, dependability
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www.cdus.ae.utexas.edu

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Thank you.

Questions?