Homework Problems

1. Show that if $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are two wavefunctions of Schrödinger equation then
\[
\frac{d}{dt} \int_{-\infty}^{\infty} \Psi_2(x,t)^* \Psi_1(x,t) = 0.
\]

2. A particle at $t = 0$ is represented by the wavefunction
\[
\psi(x) = \begin{cases} A(a^2 - x^2), & |x| < a \\ 0, & \text{otherwise} \end{cases}
\]
Find $A$. Find expected value of $x$, $x^2$, $p$, $p^2$ at $t = 0$.

3. Recall we said our problem is quantum mechanical when Debroglie wavelength $\lambda = \frac{h}{p}$ is greater than characteristic length at which potential varies. In solid state potential due to atomic nuclei varies at scale of inter atomic spacing of say $3A^\circ$. Show that at room temperatures ($300K$ electrons in solids are quantum mechanical). You may estimate the velocity of the electron from $\frac{1}{2}mv^2 = \frac{3}{2}kT$.

4. Let $\Psi(x,t)$ be solution to Schrödinger equation. Show that expected value $\langle E \rangle = \langle \Psi, H\Psi \rangle$ stays constant.

5. A particle in an infinite square potential as described has initial wavefunction
\[
\Psi(x,0) = A \sin^n \frac{\pi x}{a}, \quad 0 < x < a.
\]
Find $A$, $\Psi(x,t)$, $\langle x \rangle$ and $\langle E \rangle$.

6. Find eigen-energies for a half harmonic oscillator with potential
\[
V(x) = \frac{1}{2}kx^2, \quad x \geq 0,
\]
and $\infty$ for $x < 0$.

7. For the double well potential shown in figure, find the ground state and first excited state wavefunction $\psi_0$ and $\psi_1$ when $b = 0$, $b \sim a$ and $b \gg a$. Plot the corresponding energies $E_0(b)$ and $E_1(b)$ as function of $b$. 
8. Construct the wavefunction of hydrogen in state \( n = 4, l = 3, m = 3 \) and express \( \psi \) in terms of \( r, \theta, \phi \). Find \( \langle r \rangle \). If we measure \( L_x^2 + L_y^2 \) on this wavefunction, what values can be expect and with what probability.

9. Consider a simple cubic lattice with \( a \) and \( t \) as the lattice parameter and the hopping parameter and onsite energy \( \epsilon_0 \). Find dispersion relation \( \epsilon(k) \), where assume interaction with nearest nghbs. Assuming one electron per site, how is the band filled.

10. Consider a BCC lattice with \( a \) and \( t \) as the lattice parameter and the hopping parameter and onsite energy \( \epsilon_0 \). Find dispersion relation \( \epsilon(k) \). where assume interaction with nearest nghbs. Assuming one electron per site how is the band filled.

11. Consider a FCC lattice with \( a \) and \( t \) as the lattice parameter and the hopping parameter and onsite energy \( \epsilon_0 \). Find dispersion relation \( \epsilon(k) \), where assume interaction with nearest nghbs. Assuming one electron per site how is the band filled.

12. Consider a Silicon lattice with \( a \) and \( t \) as the lattice parameter and the hopping parameter and onsite energy \( \epsilon_0 \). Assuming four \( sp^3 \) hybridized electron per site, find dispersion relation \( \epsilon(k) \) for valence and conduction band.

13. Consider a Gallium-Arsenide lattice with \( a \) and \( t \) as the lattice parameter and the hopping parameter and onsite energy \( \epsilon_1 \) on Gallium and \( \epsilon_2 \) at Aresenic. Assuming four \( sp^3 \) hybridized electron per site find dispersion relation \( \epsilon(k) \) for valence and conduction band.

14. Consider a 1D periodic potential with lattice parameter \( a = 3A^\circ \) and \( t \) parameter at 5eV. Calculate the fermi-velocity of a half filled band.

15. Now consider a 3D periodic potential with simple cubic lattice and lattice parameter \( a = 3A^\circ \) and \( t \) parameter at 5eV. Calculate the fermi-velocity for an electron in half filled band on Fermi sphere in direction \((1,0,0)\).
16. Consider a 1D periodic potential with electrons treated in free electron approximation. With potential
\[ V = V_0 \cos^2\left(\frac{\pi x}{a}\right) \] (3)
with \( V_0 = 100\text{V} \) and lattice parameter \( a = 3\text{\AA} \). Sketch first two energy bands and find the band gap.

17. In the above find the band gap between 2\textsuperscript{nd} and 3\textsuperscript{rd} band.

18. Consider a 3D periodic potential with electrons treated in free electron approximation. With potential
\[ V = V_0 \cos^2\left(\frac{\pi x}{a}\right) \cos^2\left(\frac{\pi y}{a}\right) \cos^2\left(\frac{\pi z}{a}\right), \] (4)
with \( V_0 = 100\text{V} \) and lattice parameter \( a = 3\text{\AA} \). Find the band gap between first two energy levels.

19. In class we considered a molecular bond between two atomic orbitals each with energy \( \epsilon \) and transfer element \( -t \). This was a covalent bond. Generalize this to case when atomic orbitals have energy \( \epsilon_A \) and \( \epsilon_B \) respectively. What are the energies of two molecular orbitals bonding and antibonding. This is called an ionic bond. Like table salt, NaCl.

20. We considered tight binding in monoatomic chain. We now consider diatomic chain
\[ A - B - A - B - \cdots - A - B \]
where onsite energy at site \( A \) is \( \epsilon_A \) and site \( B \) is \( \epsilon_B \) and transfer element is \( -t \). Find the dispersion relation and sketch it for this chain.

21. A silicon ingot is doped with \( 10^{16} \) arsenic atoms/cm\(^3\). Find the carrier concentrations and the Fermi level at room temperature (300 K).

22. Calculate the inbuilt potential for a silicon pn junction with \( N_A = 10^{18}/\text{cm}^3 \) and \( N_D = 10^{15}/\text{cm}^3 \) at 300K.

23. In Fig. 5.5B of the book assume DC voltage on the base of 1V (no ac voltage). Find the collector current if the \( \beta = 100 \) and \( R_1 = 100\Omega \).

24. In above what should be \( R_2 \) for an amplifier gain to be 100.

25. Give a qualitative explanation of why the gain falls down at very high frequencies.

26. In Fig. 5.6C of the book, \( R_1 = 100\Omega \), what should be \( R_2 \) for an amplifier gain to be 100.
27. In Fig. 5.6D of the book, $R_1 = 100\Omega$, what should be $R_2$ for an amplifier gain to be 10.

28. Consider metal calcium or magnesium with two electrons in the outer shell. This says that the conduction band will be full. Why is it a metal then.

29. Consider a simple cubic lattice with $m$ as the atomic mass and $k$ as the spring constant and $a$ as the lattice parameter. Assume interaction with the nearest and second nearest neighbors. Calculate the phonon spectrum for transverse and longitudinal phonons and sketch first Brillouin zone.

30. Consider a BCC lattice with $m$ as atomic mass and $k$ as spring constant and $a$ as the lattice parameter. Assume interaction with nearest neighbors. Calculate the phonon spectrum for transverse and longitudinal phonons and sketch first Brillouin zone.

31. Consider a FCC lattice with $m$ as atomic mass and $k$ as spring constant and $a$ as the lattice parameter. Assume interaction with nearest neighbors. Calculate the phonon spectrum for transverse and longitudinal phonons and sketch first Brillouin zone.

32. A electron with fermi velocity $10^5$ m/s collides with a phonon head on, with $\theta = 90^\circ$ in the Fig. 3.16 of the book. Assuming periodic lattice potential

$$V = V_0 \cos^2\left(\frac{\pi x}{a}\right) \cos^2\left(\frac{\pi y}{a}\right) \cos^2\left(\frac{\pi z}{a}\right),$$

(5)

with $V_0 = 10V$, lattice parameter $a = 3A^0$, and Debye frequency $\omega_d = 10^{13}$ m/s. Find the temperature at which electron will back-scatter.

33. In the above problem, if temperature $T = 100K$, find the largest angle of collision $\theta$ in Fig. 3.16 of the book, for electron to back-scatter.

34. Let the current through a wire with cross section $1mm^2$ be $1Amp$. Let the relaxation time of electrons be $\tau = 10^{-14}$. What is the velocity of the electrons.

35. Consider a 3D periodic potential

$$V = V_0 \cos^2\left(\frac{\pi x}{a}\right) \cos^2\left(\frac{\pi y}{a}\right) \cos^2\left(\frac{\pi z}{a}\right),$$

(6)

with $V_0 = 10V$ and lattice parameter $a = 3A^0$ and atomic mass of 20 protons. Calculate the parameter $c$ in the book and use it to find the binding energy per electron and superconducting gap assuming debye frequency of $\omega_d = 10^{13}$ Hz and band width $\omega_F = 10$ eV.
36. In the above problem if the hopping parameter \( t = 5 \text{ eV} \), calculate \( \omega_F \) and binding energy and superconducting gap.

37. How will the superconducting gap change if the mass of the ions was doubled.

38. How will the superconducting gap change if the spring constant of the lattice was doubled.

39. How will the superconducting gap change if the lattice parameter \( a \) was doubled.

40. How will the superconducting gap change if the hopping parameter \( t \) was doubled.

41. What is minimum magnetic field allowed through a hole of radius 1 mm in a superconductor.

42. Find the paramagnetic susceptibility of conduction electrons with fermi energy at 10 eV and density \( 10^{28}/m^3 \).

43. Find the diamagnetic susceptibility of conduction electrons with fermi energy at 10 eV and density \( 10^{28}/m^3 \).

44. Neon (atomic number 10) is FCC lattice with lattice parameter \( a = 4.46 \text{Å} \). Find the diamagnetic susceptibility.

45. In quantum hall effect what is the height of the plateaus in units of \( \Omega \).

46. At a magnetic field of 10 T, what is the energy spacing between the the Landau levels.

47. In De Haas-van Alphen effect the susceptibility goes through a period as magnetic field is increased from 2.08 T to 2.1 T. What is the extremal area.

48. In the X-ray diffraction experiment, the wavelength of the X-ray is 3Å and lattice spacing 4Å. What is the smallest angle at which diffraction is seen.

49. In X-ray diffraction experiment, the wavelength of the X-ray is 3Å. If diffraction is seen at 5° and then 20° what is the lattice spacing.

50. In Neutron spectroscopy of phonons, incident neutrons at 50eV are scattered (deflected) by an angle 30° from their course. If scattered neutrons have energy 50.01eV. Find the magnitude of phonon energy and momentum.

51. In the Arpes experiment, X rays of wavelength 30Å are used. Ejected electrons are emitted at angle 45° to the vertical and azimuth 60°. If the energy of the ejected electron is 10eV, find the energy momentum of the valence electron.
52. For electrons at room temperature, what is the tunelling probability through a barrier of height 5 V and width 1 nm.