CONTROL of SWITCHED SYSTEMS with LIMITED INFORMATION

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PROBLEM FORMULATION

Switched system: $\dot{x} = A_\sigma x + B_\sigma u$

$\{(A_p, B_p) : p \in \mathcal{P}\}$ are (stabilizable) modes,

$\mathcal{P}$ is a (finite) index set, $\sigma : [0, \infty) \to \mathcal{P}$ is a switching signal

(can be state-dependent, realizing discrete state in hybrid system)

Information structure:

sampling period

Sampling: state x is measured at times $t_k = k\tau_s$, $k = 0, 1, \ldots$

Quantization: each $x(t_k)$ is encoded by an integer from 0 to N^n and sent to the controller, along with $\sigma(t_k) \in \mathcal{P}$ $\quad (n = \dim x)$

Data rate: $\dfrac{\log_2(N^n + 1) + \log_2 |\mathcal{P}|}{\tau_s}$

Objective: design an encoding & control strategy s.t. $x(t) \to 0$ based on this limited information about $x(\cdot)$ and $\sigma(\cdot)$

MOTIVATION

Switching:

- ubiquitous in realistic system models
- lots of research on stability & stabilization under switching
- tools used: common & multiple Lyapunov functions, slow switching assumptions

Quantization:

- coarse sensing (low cost, limited power, hard-to-reach areas)
- limited communication (shared network resources, security)
- theoretical interest (how much info is needed for a control task)
- tools used: Lyapunov analysis, data-rate / MATI bounds

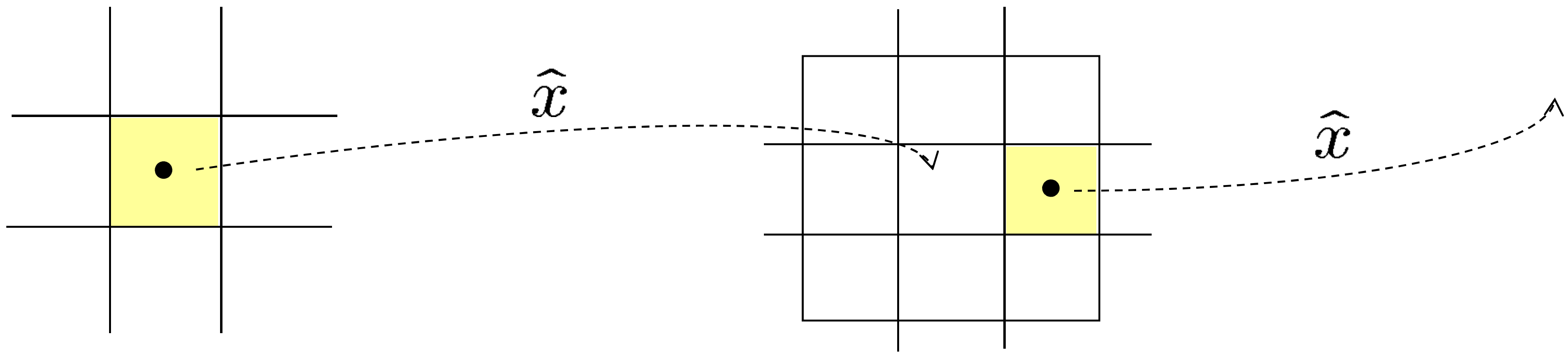
Commonality of tools is encouraging

Almost no prior work on quantized control of switched systems (except quantized MJLS [Nair et. al. 2003, Dullerud et. al. 2009])

NON-SWITCHED CASE

Quantized control of a single LTI system:

[Baillieul, Brockett-L, Hespanha, Matveev-Savkin, Nair-Evans, Tatikonda]



System can be stabilized if

$$\underbrace{\text{error reduction factor at } t_k}_{N(=3)} > \underbrace{\text{growth factor on } [t_k, t_{k+1}]}_{\text{e.g. } \|e^{A\tau_s}\|_\infty}$$

Control: $u = K\widehat{x}$

Crucial step: obtaining a reachable set over-approximation at next sampling instant

How to do this for switched systems?

REACHABLE SET ALGORITHMS

Many computational (on-line) methods for hybrid systems

- Puri–Varaiya–Borkar (1996): approximation by piecewise-constant differential inclusions; unions of polyhedra
- Henzinger–Preußig–Stursberg–et. al. (1998, 1999): approximation by rectangular automata; tools: *HyTech*, also *PHAVer* by Frehse (2005)
- Asarin–Dang–Maler (2000, 2002): linear dynamics; rectangular polyhedra; tool: *d/dt*
- Mitchell–Tomlin–et. al. (2000, 2003): nonlinear dynamics; level sets of value functions for HJB equations
- Kurzhanski–Varaiya (2002, 2005): affine open-loop dynamics; ellipsoids
- Chutinan–Krogh (2003): nonlinear dynamics; polyhedra; tool: *CheckMate*
- Girard–Le Guernic–et. al. (2005, 2008, 2009, 2011): linear dynamics; zonotopes and support functions; tool: *SpaceEx*

OUR APPROACH

We develop a method for propagating reachable set over-approximations for switched systems which is:

- Analytical (off-line)
- Leads to an a priori data-rate bound for stabilization (may be more conservative than on-line methods)
- Works with linear dynamics and hypercubes (with moving center)
- Tailored to switched systems (time-dependent switching) but can be adopted / refined for hybrid systems

SLOW-SWITCHING and DATA-RATE ASSUMPTIONS

1) $\exists$ dwell time τ_d (lower bound on time between switches)

2) $\exists$ average dwell time (ADT) τ_a s.t.

number of switches on $(s,t] \leq N_0 + \dfrac{t-s}{\tau_a} \quad \forall\, t > s \geq 0$

3) $\tau_a > \tau_d \geq \tau_s$ (sampling period)

Implies: ≤ 1 switch on each sampling interval $(t_k, t_{k+1}]$

We'll see how large τ_a should be for stability

Define $\Lambda_p := \|e^{A_p \tau_s}\|_\infty, \quad p \in \mathcal{P}$

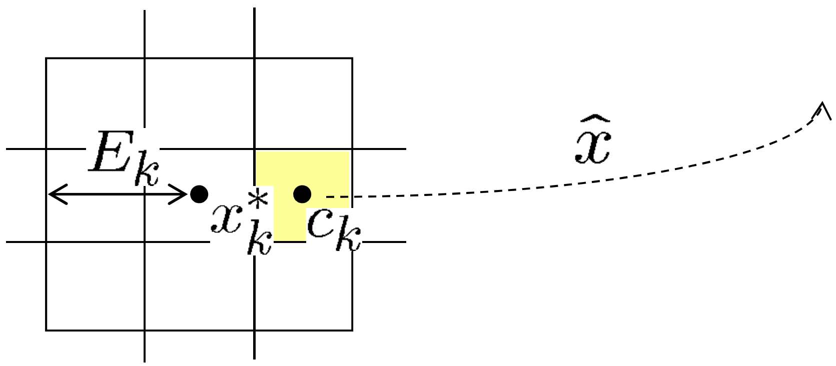
4) $\Lambda_p < N \;\; \forall p$

(usual data-rate bound for individual modes)

ENCODING and CONTROL STRATEGY

Goal: generate, on the decoder / controller side, a sequence of points $x_k^* \in \mathbb{R}^n$ and numbers $E_k > 0$ s.t.

$$\boxed{\|x(t_k) - x_k^*\| \leq E_k \ \ \forall k}$$ (always ∞-norm)



Let $p := \sigma(t_k)$

Pick K_p s.t. $A_p + B_p K_p$ is Hurwitz

Define state estimate $\widehat{x}(\cdot)$ on $[t_k, t_{k+1})$ by

$$\dot{\widehat{x}} = (A_p + B_p K_p)\widehat{x}, \qquad \widehat{x}(t_k) = c_k$$

Define control $u(\cdot)$ on $[t_k, t_{k+1})$ by

$$u(t) = K_p\,\widehat{x}(t)$$

GENERATING STATE BOUNDS

Choosing a sequence $E_0, E_1, E_2, \ldots$ that grows faster than system dynamics, for some k_0 we will have $\|x(t_{k_0})\| \le E_{k_0}$

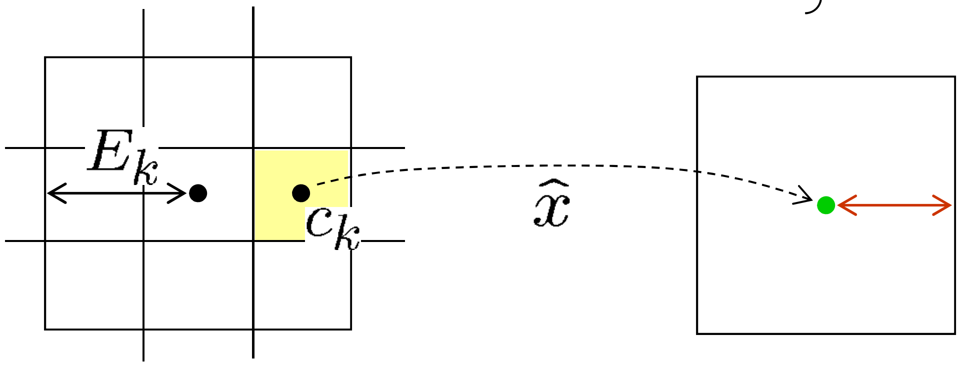
Inductively, assuming $\|x(t_k) - x_k^*\| \le E_k$ we show how to find x_{k+1}^*, E_{k+1} s.t. $\|x(t_{k+1}) - x_{k+1}^*\| \le E_{k+1}$

Case 1 (easy): sampling interval with no switch

$$\sigma(t_k) = \sigma(t_{k+1}) = p \;\Rightarrow\; \sigma(t) = p \;\forall t \in [t_k, t_{k+1}]$$

$$\left.\begin{aligned} \dot{x} &= A_p x + B_p u \\ \dot{\hat{x}} &= A_p \hat{x} + B_p u \end{aligned}\right\} \text{Let } e := x - \hat{x} \;\Rightarrow\; \dot{e} = A_p e$$

$$\left.\begin{aligned} &\dot{e} = A_p e \\ &\hat{x}(t_k) = c_k \Rightarrow \|e(t_k)\| \le E_k/N \end{aligned}\right\} \Rightarrow \|e(t_{k+1})\| \le \|e^{A_p \tau_s}\| \cdot \|e(t_k)\| \le \Lambda_p E_k / N =: E_{k+1}$$

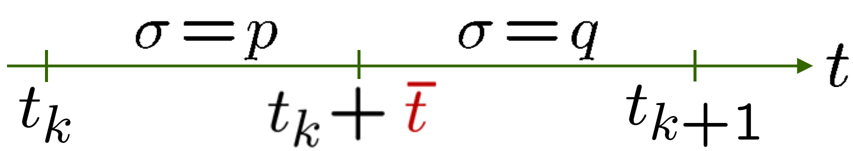


$$x_{k+1}^* := e^{(A_p + B_p K_p)\tau_s} c_k$$

GENERATING STATE BOUNDS

Case 2 (harder): sampling interval with a switch

$$\sigma(t_k) = p, \;\; \sigma(t_{k+1}) = q \neq p$$



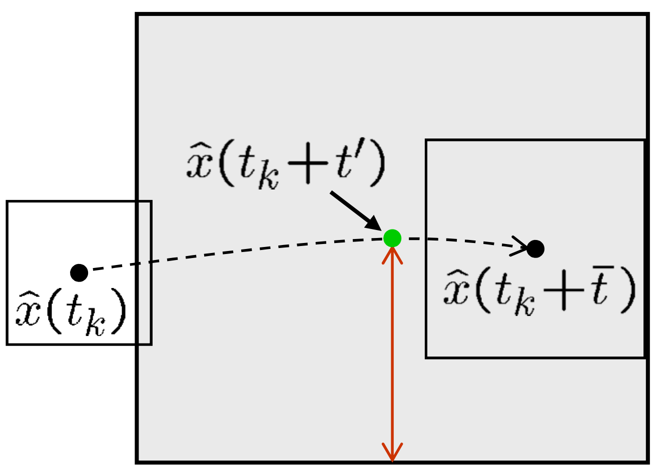
$\bar{t}$ – unknown to the controller

Before the switch: as on previous slide,

$$\|x(t_k+\bar{t}\,) - \underbrace{\hat{x}(t_k+\bar{t}\,)}\| \le \|e^{A_p\bar{t}}\|\, E_k/N$$

↖ but this is unknown ↙ known

Instead, pick some $t' \in [0, \tau_s]$ and use $\overbrace{\hat{x}(t_k + t')}$ as center



(triangle inequality)

Intermediate bound:

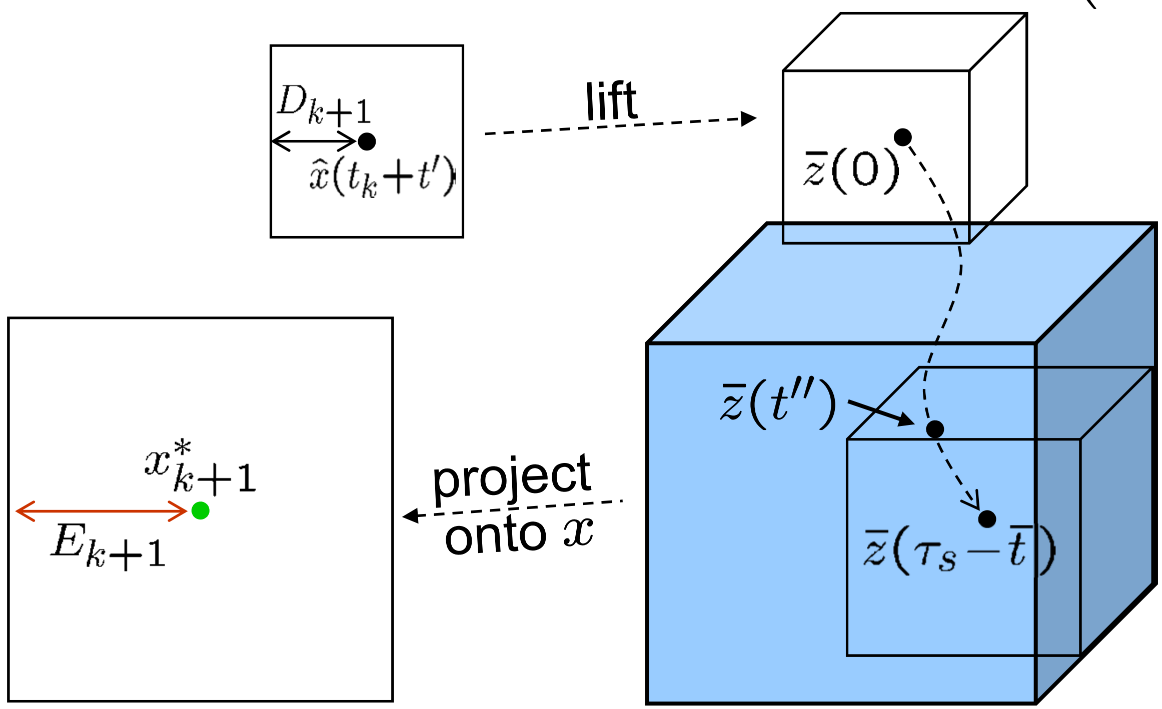
$$\|x(t_k+\bar{t}\,) - \hat{x}(t_k+t')\| \le D_{k+1}(\bar{t}\,)$$

GENERATING STATE BOUNDS

After the switch: on $[t_k+\bar{t}, t_{k+1}]$, closed-loop dynamics are

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A_q & B_q K_p \\ 0 & A_p + B_p K_p \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}, \text{ or } \dot{z} = \bar{A}_{pq} z$$

Auxiliary system in $\mathbb{R}^{2n}$: $\dot{\bar{z}} = \bar{A}_{pq}\bar{z}, \quad \bar{z}(0) = \begin{pmatrix} \hat{x}(t_k + t') \\ \hat{x}(t_k + t') \end{pmatrix}$



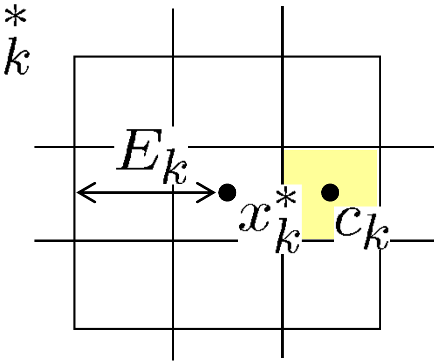
Then take maximum over $\bar{t}$ to obtain final bound

STABILITY ANALYSIS: OUTLINE

1) sampling interval with no switch: $\sigma \equiv p$ on $[t_k, t_{k+1}]$

$$x^*_{k+1} = e^{(A_p+B_pK_p)\tau_s} c_k = e^{(A_p+B_pK_p)\tau_s} (x^*_k + (c_k - x^*_k))$$

This is exp. stable DT system w. input $\Delta_k := c_k - x^*_k$



$\|\Delta_k\| \le E_k(N-1)/N$ data-rate assumption

and $E_{k+1} = E_k \Lambda_p / N < E_k \;\Rightarrow\; E_k \xrightarrow{\text{exp}} 0$

Thus, the overall "cascade" system is exp. stable

Lyapunov function: $V_p(x, E) := x^T P_p x + \rho_p E^2$

satisfies $V_p(x^*_{k+1}, E_{k+1}) \le \nu V_p(x^*_k, E_k), \;\; \nu < 1$

2) if $[t_k, t_{k+1}]$ contains a switch from p to q, then

$$V_q(x^*_{k+1}, E_{k+1}) \le \mu V_p(x^*_k, E_k), \;\; \mu > 1$$

If ADT satisfies $\tau_a > (1 + \log(\mu)/\log(1/\nu))\tau_s$ then

$V_{\sigma(t_k)}(x^*_k, E_k) \xrightarrow{\text{exp}} 0$ as $k \to \infty \;\Rightarrow\;$ same true for $x(t_k)$

Intersample bound, Lyapunov stability – see [L, Automatica, Feb'14]

SIMULATION EXAMPLE

$\mathcal{P} = \{1, 2\}$

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\; B_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\; K_1 = \begin{pmatrix} -2 & 0 \end{pmatrix}$$

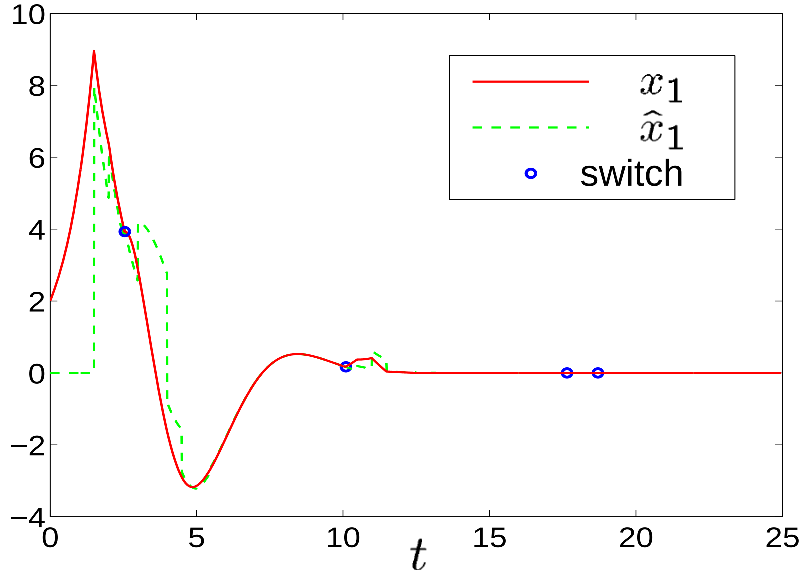
$x_0 = \binom{2}{2}$

$$A_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\; B_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\; K_2 = \begin{pmatrix} 0 & -1 \end{pmatrix}$$

$E_0 = 0.5$

$\tau_s = 0.5,\ N = 5$ (data-rate assumption holds)

$\tau_d = 1.05,\ \tau_a = 7.55,\ N_0 = 5$



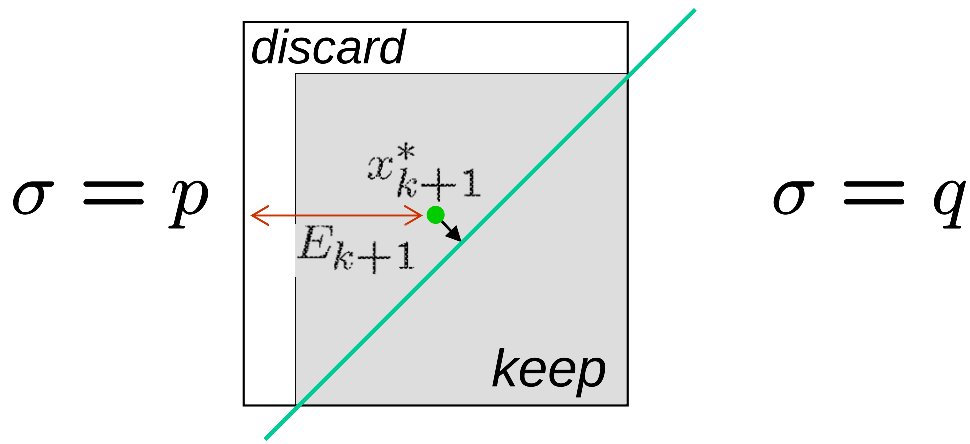
Theoretical lower bound on τ_a is about 50

HYBRID SYSTEMS

Switching triggered by switching surfaces (guards) in state space

- Previous result applies if we can use relative location of switching surfaces to verify slow-switching hypotheses
- Can just run the algorithm and verify convergence on-line
- Can use the extra info to improve reachable set bounds

For example: $\sigma(t_k) = p,\ \sigma(t_{k+1}) = q \neq p$



- State jumps – easy to incorporate

CONCLUSIONS and FUTURE WORK

Contributions:

- Stabilization of switched/hybrid systems with quantization
- Main step: computing over-approximations of reachable sets
- Data-rate bound is the usual one, maximized over modes

Extensions:

- Refining reachable set bounds (set shapes, choice of t', t'')
- Relaxing slow-switching assumptions ($\tau_d < \tau_s$)
- Less frequent transmissions of discrete mode value

Challenges:

- Output feedback (Wakaiki and Yamamoto, MTNS'14)
- External disturbances (ongoing work with Yang)
- Modeling uncertainty
- Nonlinear dynamics

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$$\dot{x} = A_\sigma x + B_\sigma u + D_\sigma d$$