Transmission Scheduling for Remote State Estimation and Control With an Energy Harvesting Sensor

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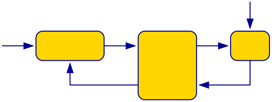
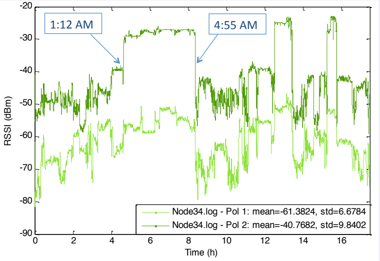


Wireless Sensor Technologies

- Due to advances in micro-electro-mechanical systems technology, small and low cost sensors with sensing, computation and wireless communication capabilities have become widely available
- Key components in wireless sensor networks, networked control systems, cyber-physical systems, Internet of Things, etc.

Energy Management

- Communication between sensors often over wireless networks
- Wireless channels are usually randomly time-varying
- Transmitted signals can be attenuated, distorted, delayed, or lost
- Transmission reliability can be improved by increasing transmission energy, but this reduces battery life $\rightarrow$ energy management

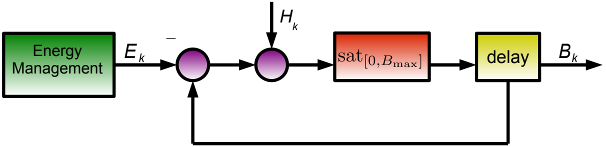


Measurements taken at Holmen's Paper Mill in Iggesund, Sweden (A. Ahlén)

Energy Harvesting

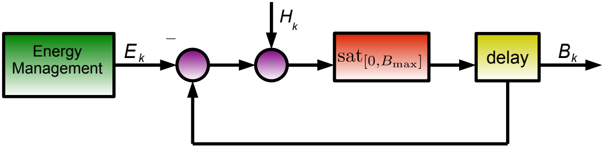
- Sensors often run on batteries which are not easily replaced
- Sensors may need to operate for years without battery change
- Energy harvesting sensors recharge their batteries by collecting energy from the environment
  - e.g. solar, thermal, mechanical vibrations
  - Potential for self-sustaining systems

Energy Harvesting



- Battery level evolves as $B_{k+1} = \min\{B_k - E_k + H_{k+1}, B_{\max}\}$, where B_k is battery level at time k, E_k is energy used at time k, H_{k+1} is energy harvested between times k and $k+1$, $B_{\max}$ is maximum battery capacity
- Key Issue: How much energy E_k should be used at time k?
  - Should we use more energy now, or save energy for later?
  - Also try to avoid battery level saturating

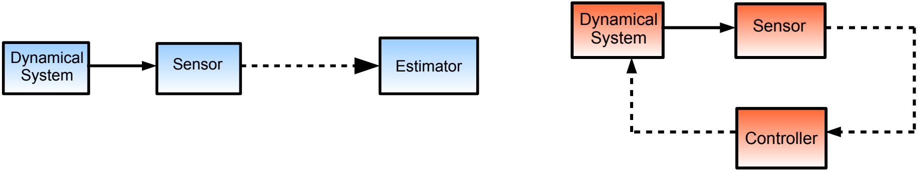
Energy Harvesting



- Energy harvesting has been studied extensively in wireless communications, e.g. maximizing throughput or minimizing transmission delay[1] [2] [3]
- Has also gained recent attention in state estimation and control, e.g. minimizing estimation error covariance[4] [5] or minimizing LQG control cost[6]

[1] Sharma, Mukherji, Joseph, Gupta, *IEEE Trans. Wireless Commun.*, 2010
[2] Ozel, Tutuncuoglu, Yang, Ulukus, Yener, *IEEE J. Sel. Areas Commun.*, 2011
[3] Ho, Zhang, *IEEE Trans. Signal Process.*, 2012
[4] Nourian, Leong, Dey, *IEEE Trans. Automat. Control*, 2014
[5] Li, Zhang, Quevedo, Lau, Dey, Shi, *IEEE Trans. Automat. Control*, 2017
[6] Knorn, Dey, *Automatica*, 2017

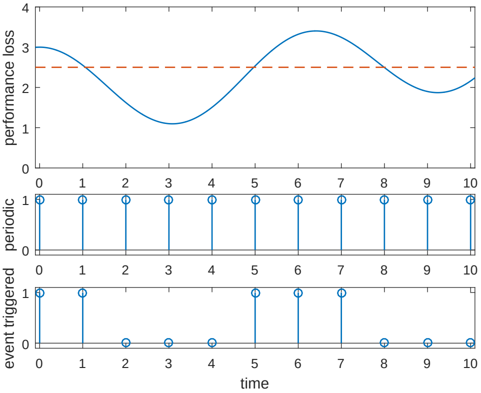
Event Triggered Estimation and Control



- Traditionally in estimation and control, measurements and control signals are transmitted periodically
- Event Triggered View - Transmit only when certain events occur, e.g. if system performance has deteriorated by a large amount
- Event triggering can achieve energy savings
- Event triggered estimation and control has been studied by Åström, Başar, Dimarogonas, Heemels, Hespanha, Hirche, Johansson, Lemmon, Shi, Tabuada, Trimpe, Wu, ...

Event Triggered Estimation and Control

- Different transmission strategies have been studied
- Threshold policies often proposed



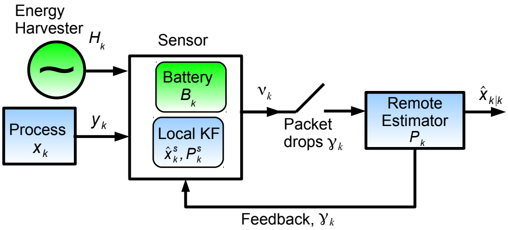
Key Questions

What are good transmission policies for remote state estimation using wireless sensors with energy harvesting capabilities?

What is the role of event triggered methods?

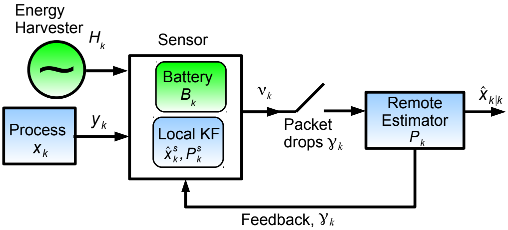
Outline

Remote State Estimation



- Process $x_{k+1} = Ax_k + w_k, \quad w_k \sim N(0, Q)$
- Sensor measurement $y_k = Cx_k + v_k, \quad v_k \sim N(0, R)$
- Sensor runs a local Kalman filter to compute (posterior) local estimates $\hat{x}^s_k$
- Local estimates transmitted over i.i.d. packet dropping link

Local Sensor Computations



- (Local) State estimates

$$\hat{x}^s_{k|k-1} \triangleq \mathbb{E}[x_k | y_0, \dots, y_{k-1}],$$
$$\hat{x}^s_k \triangleq \mathbb{E}[x_k | y_0, \dots, y_k]$$

- (Local) Estimation error covariances

$$P^s_{k|k-1} \triangleq \mathbb{E}[(x_k - \hat{x}^s_{k|k-1})(x_k - \hat{x}^s_{k|k-1})^T | y_0, \dots, y_{k-1}]$$
$$P^s_k \triangleq \mathbb{E}[(x_k - \hat{x}^s_{k|k})(x_k - \hat{x}^s_{k|k})^T | y_0, \dots, y_k]$$

Local Sensor Computations

- State estimates and error covariances are computed using the Kalman filter

$$\begin{aligned}\hat{x}^s_{k+1|k} &= A\hat{x}^s_k \\ \hat{x}^s_k &= \hat{x}^s_{k|k-1} + K_k(y_k - C\hat{x}^s_{k|k-1}) \\ P^s_{k+1|k} &= AP^s_k A^T + Q \\ P^s_k &= P^s_{k|k-1} - P^s_{k|k-1}C^T(CP^s_{k|k-1}C^T + R)^{-1}CP^s_{k|k-1}\end{aligned}$$

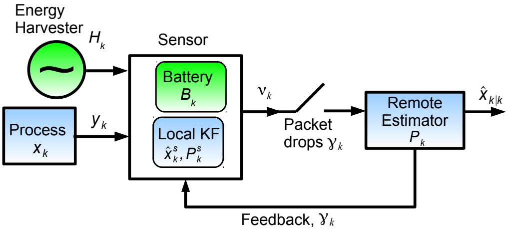
where

$$K_k = P^s_{k|k-1}C^T(CP^s_{k|k-1}C^T + R)^{-1}$$

- Under standard assumptions[7], $P^s_k \to \bar{P}$ as $k \to \infty$

[7] (A, C) observable and $(A, Q^{1/2})$ controllable

Sensor Transmissions



- Transmission decisions: Sensor transmits local state estimate to remote estimator if $\nu_k = 1$, doesn't transmit if $\nu_k = 0$

Transmitting local state estimates gives better performance over packet dropping link than transmitting measurements[a], as local estimate captures all relevant information when received

[a] Xu, Hespanha, *Proc. CDC*, 2005

- Packet drop process i.i.d. Bernoulli with $\gamma_k = 1$ if transmission successful, $\gamma_k = 0$ otherwise

Remote Estimator

- In the presence of dropouts, the information available to the remote estimator at time k is

$$\mathcal{I}_k \triangleq \{\nu_0, \ldots, \nu_k, \nu_0\gamma_0, \ldots, \nu_k\gamma_k, \nu_0\gamma_0\hat{x}_0^s, \ldots, \nu_k\gamma_k\hat{x}_k^s\}$$

- Define remote state estimates and estimation error covariances

$$\hat{x}_k \triangleq \mathbb{E}[x_k|\mathcal{I}_k], \; P_k \triangleq \mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T|\mathcal{I}_k].$$

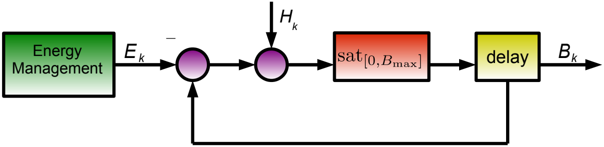
- Remote estimator has the form

$$\hat{x}_k = \begin{cases} A\hat{x}_{k-1} & , \quad \nu_k\gamma_k = 0 \\ \hat{x}_k^s & , \quad \nu_k\gamma_k = 1 \end{cases}$$

$$P_k = \begin{cases} AP_{k-1}A^T + Q & , \quad \nu_k\gamma_k = 0 \\ \bar{P} & , \quad \nu_k\gamma_k = 1 \end{cases}$$

When transmission received, update remote estimate as local estimate. When transmission is not received, use one step ahead prediction

Energy Management



- Transmission decisions: Sensor transmits local state estimate if $\nu_k = 1$, doesn't transmit if $\nu_k = 0$
- Each transmission uses energy E
- Battery level evolves as

$$\begin{aligned} B_{k+1} &= \min\{B_k - E_k + H_{k+1}, B_{\max}\} \\ &= \min\{B_k - \nu_k E + H_{k+1}, B_{\max}\} \end{aligned}$$

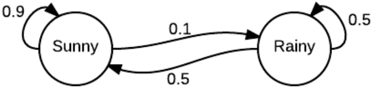
- Harvested energy process $\{H_k\}$ is Markov

Energy Management

- Harvested energy process $\{H_k\}$ is Markov, to model time correlations in amount of energy harvested
- Example 1. For solar energy, very little/no energy can be harvested at night
- Example 2. Suppose the weather X_n on day n is either sunny (state 1) or rainy (state 2), and is modelled as a Markov chain with transition probabilities

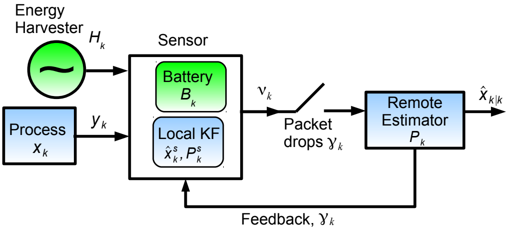
$$\mathbf{P} = \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix},$$

with the (i, j)-th entry of **P** representing $\mathbb{P}(X_{n+1} = j | X_n = i)$



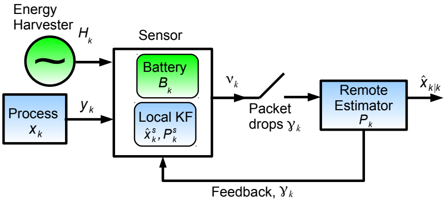
Outline

Transmission Scheduling



- Battery level evolves as $B_{k+1} = \min\{B_k - \nu_k E + H_{k+1}, B_{\max}\}$
- Key Question: Should we transmit now, or save energy for later?

Optimal Transmission Scheduling



- Determine the transmission schedule that minimizes the expected error covariance at remote estimator

$$\min_{\{\nu_1,\ldots,\nu_K\}} \sum_{k=1}^{K} \mathbb{E}[\mathrm{tr} P_k]$$

  subject to energy harvesting constraints

$$\nu_k E \leq B_k, \forall k,$$

  with battery dynamics $B_{k+1} = \min\{B_k - \nu_k E + H_{k+1}, B_{\max}\}$
- Decision variables ν_k depend on (P_{k-1}, H_k, B_k)

Optimal Transmission Scheduling

$$\min_{\{\nu_1,\ldots,\nu_K\}} \sum_{k=1}^{K} \mathbb{E}[\mathrm{tr} P_k]$$

subject to

$$\nu_k E \leq B_k, \forall k, \quad B_{k+1} = \min\{B_k - \nu_k E + H_{k+1}, B_{\max}\},$$

where decision variables ν_k depend on (P_{k-1}, H_k, B_k)

- Problem can be solved numerically using dynamic programming
- However dynamic programming doesn't provide much insight into the form of the optimal solution
- We will analyze the problem further to derive structural results
  - This leads to insights and computational savings

Structural Properties of Optimal Schedule

Theorem

(i) For fixed B_k and H_k, the optimal ν_k^* is a threshold policy on P_{k-1} of the form:

$$\nu_k^*(P_{k-1}, B_k, H_k) = \begin{cases} 0 & , \quad P_{k-1} \leq P_k^* \\ 1 & , \quad \text{otherwise} \end{cases}$$

where the threshold P_k^* depends on k, B_k and H_k.

For large P_{k-1}, it is better to transmit than not transmit

Idea of proof: Show that the difference in expected cost between transmitting and not transmitting is monotonic in P_{k-1} (when B_k and H_k are fixed)

- Use an induction argument to prove this

Structural Properties of Optimal Schedule

Theorem

(ii) For fixed P_{k-1} and H_k, the optimal ν_k^* is a threshold policy on B_k of the form:

$$\nu_k^*(P_{k-1}, B_k, H_k) = \begin{cases} 0 & , \quad B_k \leq B_k^* \\ 1 & , \quad \text{otherwise} \end{cases}$$

where the threshold B_k^* depends on k, P_{k-1} and H_k.

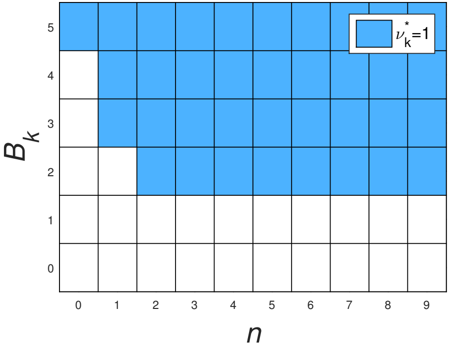
More likely to transmit when battery level is high

Idea of proof: Show that the value functions of dynamic programming algorithm, when regarded as a function of B_k and ν_k, are submodular in (B_k, ν_k). This then implies[8] that ν_k^* is non-decreasing with P_{k-1}.

[8] Topkis, *Operations Research*, 1978

Structural Properties of Optimal Schedule

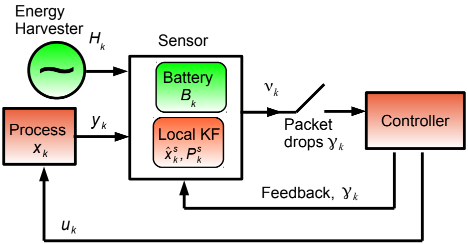
- Optimal policies are of threshold-type, event based
  - simplifies real-time implementation
  - can also provide computational savings in numerical solution



  - $P_{k-1} = f^n(\bar{P})$, where $f(\bar{P}) \triangleq A^T \bar{P} A + Q$

Outline

Transmission Scheduling for Control



- Can also study the control problem
- System model similar to estimation problem, except process is now

$$x_{k+1} = Ax_k + \mathfrak{B}u_k + w_k$$

Transmission Scheduling for Control

- Equations for local Kalman filter are now

$$
\begin{aligned}
\hat{x}^s_{k+1|k} &= A\hat{x}^s_k + \mathfrak{B}u_k \\
\hat{x}^s_k &= \hat{x}^s_{k|k-1} + K_k(y_k - C\hat{x}^s_{k|k-1}) \\
P^s_{k+1|k} &= AP^s_k A^T + Q \\
P^s_k &= P^s_{k|k-1} - P^s_{k|k-1}C^T(CP^s_{k|k-1}C^T + R)^{-1}CP^s_{k|k-1}
\end{aligned}
$$

where

$$K_k = P^s_{k|k-1}C^T(CP^s_{k|k-1}C^T + R)^{-1}$$

- Note that u_k can be reconstructed at sensor from γ_k, since $\hat{x}_k$ can be reconstructed from γ_k, and optimal u_k will be a linear function of $\hat{x}_k$ (see later)

Transmission Scheduling for Control

- Want to solve the following problem

$$\min_{\substack{\{\nu_1,\dots,\nu_K,\\ u_1,\dots,u_K\}}} \mathbb{E}\left[\sum_{k=1}^{K}(x_k^T W x_k + u_k^T U u_k) + x_{K+1}^T W x_{K+1}\right]$$

subject to energy harvesting constraints

$$\nu_k E \leq B_k, \forall k$$

- Is a joint control and scheduling problem
- For transmission decisions ν_k dependent on (P_{k-1}, B_k, H_k), problem can be shown to be separable, and is equivalent to

$$\min_{\{\nu_1,\dots,\nu_K\}}\left[\min_{\{u_1,\dots,u_K\}} \mathbb{E}\left[\sum_{k=1}^{K}(x_k^T W x_k + u_k^T U u_k) + x_{K+1}^T W x_{K+1}\right]\right]$$

$$\min_{\{\nu_1,\dots,\nu_K\}} \Big[\min_{\{u_1,\dots,u_K\}} \mathbb{E}\Big[\sum_{k=1}^{K} (x_k^T W x_k + u_k^T U u_k) + x_{K+1}^T W x_{K+1} \Big]\Big]$$

- Inner optimization is LQG-type problem with solution

$$\begin{aligned}
u_k^* &= -(\mathfrak{B}^T S_{k+1} \mathfrak{B} + U)^{-1} \mathfrak{B}^T S_{k+1} A \hat{x}_k, \\
S_{K+1} &= W, \\
S_k &= A^T S_{k+1} A + W - A^T S_{k+1} \mathfrak{B} (\mathfrak{B}^T S_{k+1} \mathfrak{B} + U)^{-1} \mathfrak{B}^T S_{k+1} A
\end{aligned}$$

- Optimal cost is

$$\operatorname{tr}(S_1 P_1) + \sum_{k=1}^{K} \operatorname{tr}(S_{k+1} Q) + \sum_{k=1}^{K} \operatorname{tr}\big((A^T S_{k+1} A + W - S_k)\mathbb{E}[P_k]\big)$$

$$\min_{\{\nu_1,\dots,\nu_K\}} \left[\min_{\{u_1,\dots,u_K\}} \mathbb{E}\left[\sum_{k=1}^{K} (x_k^T W x_k + u_k^T U u_k) + x_{K+1}^T W x_{K+1} \right]\right]$$

- Substituting optimal cost of inner optimization

$$\mathrm{tr}(S_1 P_1) + \sum_{k=1}^{K} \mathrm{tr}(S_{k+1} Q) + \sum_{k=1}^{K} \mathrm{tr}\big((A^T S_{k+1} A + W - S_k)\mathbb{E}[P_k]\big),$$

  the following transmission scheduling problem remains:

$$\min_{\{\nu_1,\dots,\nu_K\}} \left[\sum_{k=1}^{K} \mathrm{tr}\big((A^T S_{k+1} A + W - S_k)\mathbb{E}[P_k]\big) \right],$$

  subject to energy harvesting constraint $\nu_k E \leq B_k, \forall k$
- Similar to transmission scheduling problem for remote estimation discussed before

Theorem

In the transmission scheduling problem for control:
(i) For fixed B_k and H_k, the optimal ν_k^* is a threshold policy on P_{k-1} of the form:

$$\nu_k^*(P_{k-1}, B_k, H_k) = \begin{cases} 0 & , \quad P_{k-1} \leq \tilde{P}_k^* \\ 1 & , \quad \text{otherwise} \end{cases}$$

where the threshold $\tilde{P}_k^*$ depends on k, B_k and H_k.
(ii) For fixed P_{k-1} and H_k, the optimal ν_k^* is a threshold policy on B_k of the form:

$$\nu_k^*(P_{k-1}, B_k, H_k) = \begin{cases} 0 & , \quad B_k \leq \tilde{B}_k^* \\ 1 & , \quad \text{otherwise} \end{cases}$$

where the threshold $\tilde{B}_k^*$ depends on k, P_{k-1} and H_k.

Outline

Simulation Studies

- Parameters

$$A = \begin{bmatrix} 1.2 & 0.2 \\ 0.2 & 0.7 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \; Q = I, \; R = 1$$

- Packet reception probability $\lambda = 0.7$, transmission energy $E = 2$
- Harvested energy process $\{H_k\}$ is Markov with state space $\{0, 1, 2\}$ and transition probability matrix[9]

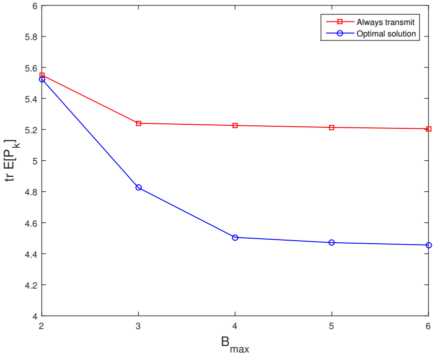
$$\mathbf{P} = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

- Horizon $K = 10$.

[9] Energy is scarce in this example

Simulation Studies

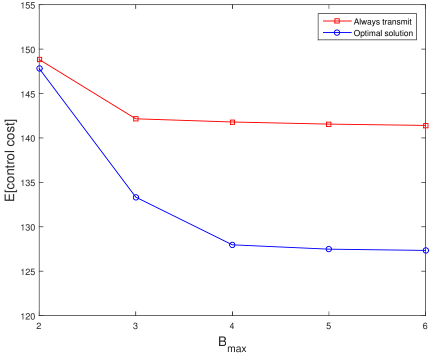
- Estimation problem. Comparison with greedy method which always transmits provided there is enough energy in battery



- The optimal solution outperforms greedy method, without using more energy

Simulation Studies

- Control problem. Same parameters as estimation problem, plus $\mathfrak{B} = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$, $W = I$, $U = 1$.
- Comparison with greedy method which always transmits provided there is enough energy, together with optimal LQG controller



Conclusion

- Energy harvesting introduces new design issues
- We have studied transmission scheduling problems for remote state estimation and control with an energy harvesting sensor
- We showed that threshold policies are optimal

Open Questions

- Derive structural results for
  - Power control instead of transmission scheduling
  - Multiple sensors
- Wireless power transfer and energy sharing
  - Transfer of electrical energy without wires using electro-magnetic (EM) fields and EM radiation
  - Both near field (e.g. wireless phone chargers) and far field (over km distances) techniques currently under active investigation
- Energy harvesting from ambient EM waves also being investigated

Further Reading

The current presentation is based on:

- Leong, Dey, Quevedo, “Transmission Scheduling for Remote State Estimation and Control With an Energy Harvesting Sensor”, to be published in *Automatica*

Other related work:

- Li, Zhang, Quevedo, Lau, Dey, Shi, “Power Control of an Energy Harvesting Sensor for Remote State Estimation”, *IEEE Transactions on Automatic Control*, January 2017
- Leong, Quevedo, Dey, “Optimal Control of Energy Resources for State Estimation Over Wireless Channels”, *Springer*, 2018