Feedback Tracking Control Schemes For A Class of Underactuated Vehicles in $\mathrm{SE}(3)$

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Autonomous Vehicles

- Autonomous UAV : Underactauted:
- Six degrees of freedom
- Four degrees of actuation
  - Torque can be applied about any body-fixed coordinate axes
  - Thrust fixed along a single body-fixed axis
  - Only the magnitude of thrust is controlled

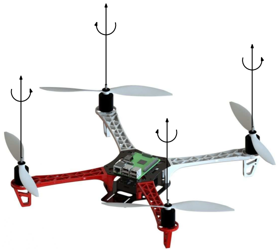

Figure: Example: Quadcopter / quadrotor

Coordinate frame definition

- $\mathcal{B}$ is the body fixed frame, fixed to the CoM of the vehicle
- $\mathcal{I}$ is fixed in space and takes the role of an inertial coordinate frame

Thus the pose of the vehicle in $\mathrm{SE}(3)$ given by,

$$g = \begin{bmatrix} R & b \\ 0 & 1 \end{bmatrix} \in \mathrm{SE}(3) \tag{1}$$

- $R \in \mathrm{SO}(3)$ is rotation matrix from frame $\mathcal{B}$ to frame $\mathcal{I}$
- $b \in \mathbb{R}^3$ is the position vector of the origin of frame $\mathcal{B}$ with respect to frame $\mathcal{I}$ represented in frame $\mathcal{I}$

Kinematic Equations

Pose kinematics of the vehicle is given by

$$\dot{R} = R(\Omega)^\times,$$
$$\dot{b} = v.$$

Here,

- $\Omega(t) \in \mathbb{R}^3$ is the angular velocity measured in the body frame
- $v(t) \in \mathbb{R}^3$ is the translational velocity measured in the inertial frame
- $(.)^\times : \mathbb{R}^3 \to \mathfrak{so}(3)$ is the skew-symmetric cross product operator

Dynamics Model

- Dynamics model for the underactuated vehicle is given by,

$$m\dot{v} = m\ g\ e_3 - f\ R\ e_3,$$
$$J\dot{\Omega} = J\ \Omega \times \Omega + \tau.$$

Here,

- g is acceleration due to gravity
- $e_3 = [0\ ,0\ ,1]^\top$
- $-R\ e_3$ is the inertial direction of the thrust
- $f \in \mathbb{R}$ is the thrust magnitude
- $J \in \mathbb{R}^{3\times 3}$ is the moment of inertia
- $\tau \in \mathbb{R}^3$ is the torque

Quadcopter Actuation Model

- VTOL quadcopter UAV model is considered here
  - Four identical actuators (propellers)
  - D is the scalar distance from the propeller axis to the center of UAV
  - $u \in \mathcal{C} \subset \mathbb{R}^4$ is the control input which directly actuates the 3 rotational DoF and 1 translational DoF
- Actuation model of quadcopter is given by

$$u = \begin{bmatrix} -f & \tau \end{bmatrix}^\mathsf{T} = \mathcal{K} \begin{bmatrix} \bar{\omega}_1^2 & \bar{\omega}_2^2 & \bar{\omega}_3^2 & \bar{\omega}_4^2 \end{bmatrix}^\mathsf{T},$$

where

$$\mathcal{K} = \begin{bmatrix} -k_f & -k_f & -k_f & -k_f \\ 0 & -k_f D & 0 & k_f D \\ k_f D & 0 & -k_f D & 0 \\ -k_\tau & k_\tau & -k_\tau & k_\tau \end{bmatrix} \in \mathbb{R}^{4\times 4}$$

is a constant invertible matrix for $k_f \neq 0$ and $k_\tau \neq 0$

Integrated Trajectory Generation and Control

- Off-line
  - Inspect the environment
  - Initialize the UAV
  - Select desired waypoints in $\mathbb{R}^3$ w.r.t $\mathcal{I}$
  - Generate a desired trajectory $b^d(t) \in \mathbb{R}^3$ is $\mathbb{C}^2$
- On-line
  - Compute control thrust f and the desired angular velocity Ω^d
  - Compute the attitude control torque
- UAV tracking trajectory

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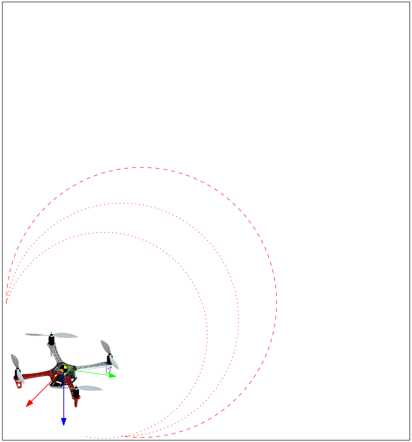

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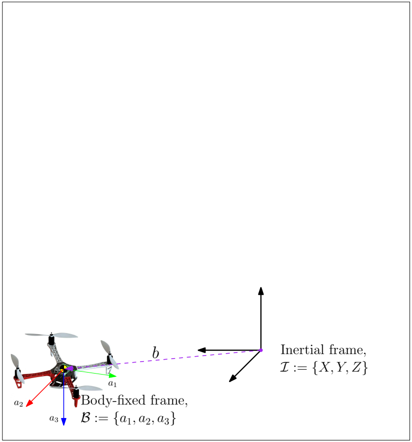

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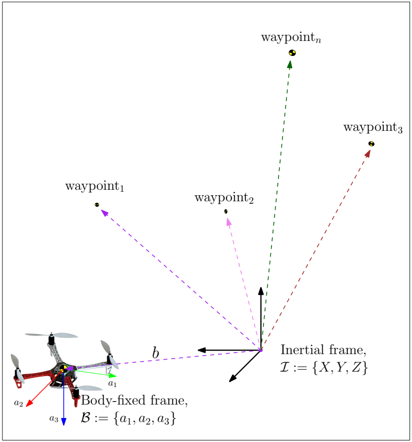

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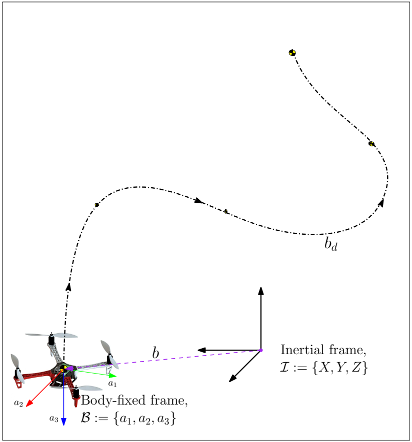

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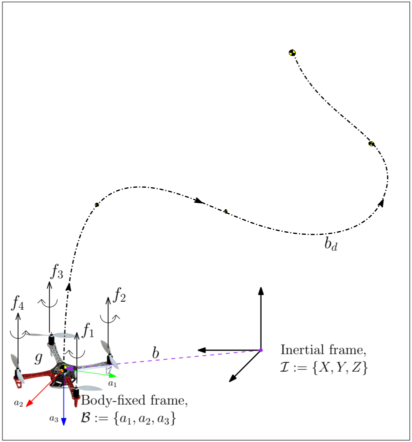

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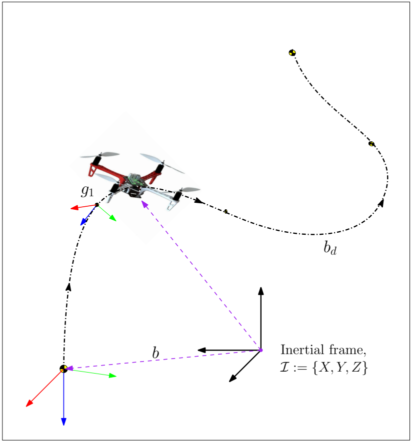

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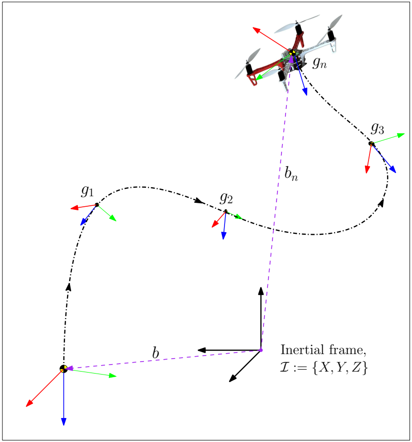

Translational Trajectory Tracking

- Position tracking error: $\tilde{b} := b - b^d$
- Translational velocity tracking error: $\tilde{v} := v - v^d = \dot{\tilde{b}}$
- Acceleration tracking error: $\tilde{a} := \dot{\tilde{v}}$
- Translational error dynamics in inertial frame $\mathcal{I}$ are:

$$\dot{\tilde{b}} = \tilde{v} = v - v^d, \quad m\dot{\tilde{v}} = mge_3 - fr_3 - m\dot{v}^d, \tag{2}$$

- After differentiating (2) and reformulating (for $\alpha > 0$),

$$\begin{cases} m\frac{d}{dt}\left(\tilde{a} + \alpha\tilde{v}\right) & = -m\left(\ddot{v}^d + \alpha\dot{v}^d\right) + \alpha\ m\ g\ e_3 - (\dot{f} + \alpha f)Re_3 \\ & \quad -f\ R(\Omega \times e_3) \end{cases}$$

- Fully actuated with controls $\dot{f}$ and $\left[\Omega_{(1)},\ \Omega_{(2)},\ 0\right]^{\mathsf{T}}$

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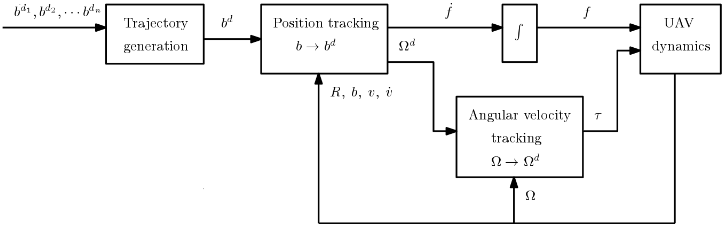
Control Framework



- Position tracking control. Design $\dot{f}$ and $\left[\Omega^1,\ \Omega^2,\ 0\right]^\mathsf{T}$ for tracking the desired position trajectory
- Angular velocity tracking control. Synthesize control torque τ so that angular velocity tracks the angular velocity profile obtained from the position tracking control design

Position Trajectory Tracking Control

Theorem

Let the feedback control law be given by:

$$\begin{aligned}\dot{f} =& -\alpha f + mR^{\mathsf{T}}\big(\zeta(\tilde{a}+\alpha\tilde{v}) + \lambda(\tilde{v}+\alpha\tilde{b})\big)\cdot e_3 \\ & - mR^{\mathsf{T}}\left(\ddot{v}^d + \alpha\dot{v}^d\right)\cdot e_3 + (\alpha mgR^{\mathsf{T}}e_3)\cdot e_3 \\ \left[\Omega_{(1)},\Omega_{(2)},0\right]^{\mathsf{T}} =& \left(\frac{1}{f}\right)e_3\times\left(mR^{\mathsf{T}}\big(\zeta(\tilde{a}+\alpha\tilde{v}) + \lambda(\tilde{v}+\alpha\tilde{b})\big)\right) \\ & - \left(\frac{1}{f}\right)e_3\times\left(mR^{\mathsf{T}}(\ddot{v}^d - \alpha\dot{v}^d) + \alpha mgR^{\mathsf{T}}e_3\right)\end{aligned}$$

where $\zeta,\ \lambda\in\mathbb{R}^+$ satisfying $\zeta^2\neq 4\lambda$. Then tracking error dynamics is stabilized to $(\tilde{b},\tilde{v},\tilde{a}) = (\mathbf{0}_{1\times 3},\mathbf{0}_{1\times 3},\mathbf{0}_{1\times 3})$ exponentially.

Position Trajectory Tracking Control

Exponential stability of position tracking control shown by expressing the feedback dynamics for $\tilde{b}$ as:

$$\left(\frac{d^3}{dt^3} + (\alpha + \zeta)\frac{d^2}{dt^2} + (\zeta\alpha + \lambda)\frac{d}{dt} + (\alpha\lambda)\right)\tilde{b} = \mathbf{0}_{1\times 3}, \qquad (3)$$

which has non-repeating roots with negative real parts.

Angular Velocity Tracking Control

- Angular velocity Ω is controlled to achieve stable tracking of desired translational motion
- Desired angular velocity $\Omega^d(t)$ chosen to be

$$\Omega^d = \left(\frac{1}{f}\right) e_3 \times \left(mR^\mathsf{T}\left(\zeta(\tilde{a} + \alpha\tilde{v}) + \lambda(\tilde{v} + \alpha\tilde{b})\right)\right) - \left(\frac{1}{f}\right) e_3 \times \left(mR^\mathsf{T}(\ddot{v}^d - \alpha\dot{v}^d) + \alpha mgR^\mathsf{T} e_3\right)$$

- The third component of angular velocity chosen to be zero
- Angular velocity error defined as $e_\Omega = \Omega - \Omega^d$
- The error dynamics are

$$\frac{d}{dt}(Je_\Omega) = J\Omega \times \Omega + \tau - J\frac{d}{dt}(\Omega^d)$$

Finite-Time Stable Angular Velocity Tracking Control

Theorem

The feedback control law τ for angular velocity control is given by

$$\tau = -(J\Omega) \times \Omega + J\frac{d}{dt}(\Omega^d) - \frac{L(\Omega - \Omega^d)}{\left((\Omega - \Omega^d)^{\mathsf{T}} L(\Omega - \Omega^d)\right)^{1-\frac{1}{p}}}$$

where L is a positive definite matrix and $p \in (1, 2)$. Then, the angular velocity error dynamics is stabilized to $e_\Omega = \mathbf{0}_{1\times 3}$ in finite time.

- Finite-time stability of the angular velocity tracking control scheme is shown by the Lyapunov function:

$$\mathcal{V}_{rot} = \frac{1}{2}(\Omega - \Omega^d)^{\mathsf{T}} J(\Omega - \Omega^d) \tag{4}$$

Finite-Time Stable Angular Velocity Tracking Control

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Stability of the Overall Feedback System

Theorem

The overall feedback system given by translational tracking error dynamics and the angular velocity error dynamics is exponentially stabilized to $(\tilde{b}, \tilde{v}, \tilde{a}, e_\Omega) = (\mathbf{0}_{1\times 3}, \mathbf{0}_{1\times 3}, \mathbf{0}_{1\times 3}, \mathbf{0}_{1\times 3})$

Overall stability of system is shown by: (1) finite-time convergence and stability of angular velocity to desired angular velocity profile; and (2) exponential convergence of position tracking errors to zero thereafter.

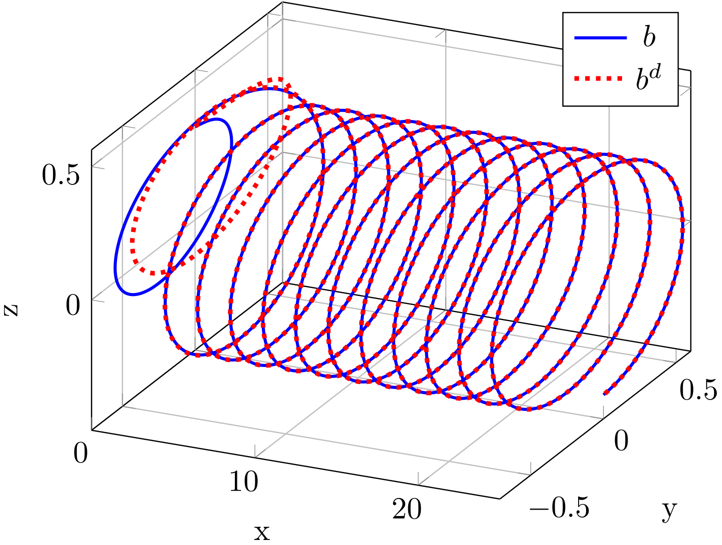
Numerical Simulations

- Moment of inertia and mass of UAV are selected as $J = \mathrm{diag}([0.0820, 0.0845, 0.1377])\mathrm{kg\ m}^2, \quad m = 4.34\mathrm{kg}.$
- Initial configuration:

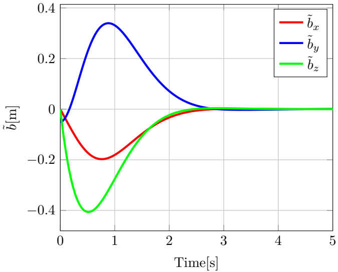
$$\begin{aligned} b(0) = [0,\ 0.35,\ 0]^{\mathsf{T}}\ \mathrm{m}, \quad v(0) = [0,\ 0,\ 0]^{\mathsf{T}}\ \mathrm{m/s} \\ R(0) = I, \quad \Omega(0) = [0,\ 0,\ 1]^{\mathsf{T}}\ \mathrm{rad/s}. \end{aligned}$$

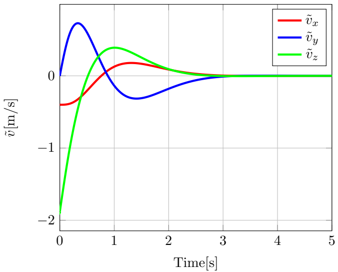
- At least two inertial sensors including three-axis rate gyro are assumed to be onboard the rigid body.
- The desired trajectory is chosen to be helical, $b^d(t) = [0.4t,\ 0.4\cos(t),\ 0.6\sin(t)]^{\mathsf{T}}\ \mathrm{m}$

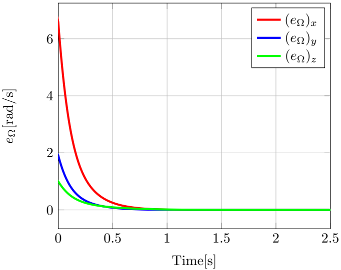
Desired and achieved position trajectories of the simulated vehicle



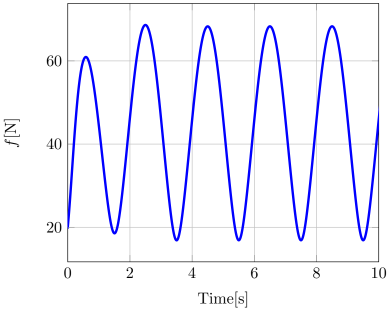
Tracking Errors

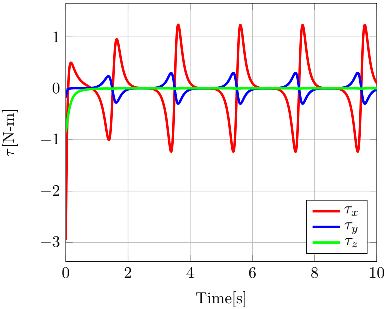






Control Inputs





Conclusion

- Trajectory generation
- Exponentially stable position trajectory tracking
- Finite time stable angular velocity tracking
- Yaw rate stabilized to zero
- Validation of integrated guidance and control

Future Work

- Experimental Results


Thank you!

Integrated Guidance and Nonlinear Feedback Control of Underactuated Unmanned Aerial Vehicles in SE(3)

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Unmanned Vehicle Systems

- Autonomous Multirotor UAV

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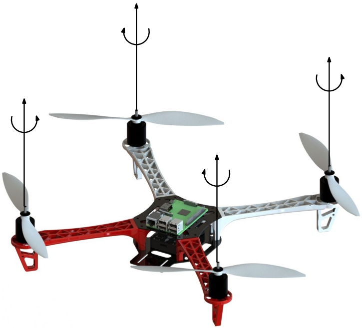
☞ Autonomous Multirotor UAV
Underactauted:



Quadrotor/Quadcopter

Quadcopter

- Four independent control inputs
- Six Degrees of Freedom (DoF); 3 DoF attitude is controlled
- Only the magnitude of thrust (1 DoF in translation) is controlled

Coordinate frame definition

- $\mathcal{B}$ is a body fixed frame, fixed to the mass center of the vehicle
- $\mathcal{I}$ is fixed in space and takes the role of an inertial coordinate frame

The pose of the vehicle is represented in matrix form as follows:

$$g = \begin{bmatrix} R & b \\ 0 & 1 \end{bmatrix} \in \mathrm{SE}(3) \tag{1}$$

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- $b \in \mathbb{R}^3$ denotes position vector of origin of frame $\mathcal{B}$ with respect to frame $\mathcal{I}$ represented in frame $\mathcal{I}$

Kinematic Equations

Pose kinematics of the vehicle is given by,

$$\dot{g}(t) = g(t)\xi(t)^{\vee}, \tag{2}$$

$$\text{with } \xi^{\vee} = \begin{bmatrix} \Omega^{\times} & \nu \\ 0 & 0 \end{bmatrix} \in \mathfrak{se}(3) \subset \mathbb{R}^{4\times 4} \text{ for } \xi = \begin{bmatrix} \Omega \\ \nu \end{bmatrix} \in \mathbb{R}^{6}.$$

Here,

- $\Omega(t) \in \mathbb{R}^3$ is the angular velocity
- $\nu(t) \in \mathbb{R}^3$ is the translational velocity
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Dynamics Model

✍ "Nominal" model of the dynamics for the underactuated vehicle is given by

$$\mathbb{I}\dot{\xi} = \mathrm{ad}^*_{\xi}\mathbb{I}\xi + \varphi(g,\xi) + Bu, \ u \in \mathcal{C} \subset \mathbb{R}^4, \tag{3}$$

where,

$$\mathbb{I} = \begin{bmatrix} J & 0_{3\times 3} \\ 0_{3\times 3} & mI_3 \end{bmatrix} \in \mathbb{R}^{6\times 6}, \ B = \begin{bmatrix} I_3 & 0_{3\times 1} \\ 0_{3\times 3} & e_3 \end{bmatrix} \in \mathbb{R}^{6\times 4},$$

- ☛ $\mathbb{I}$ denotes the mass (m) and inertia (J) properties of the vehicle
- ☛ $\varphi(g,\xi) \in \mathbb{R}^6$ is the vector of known (modeled) moments and forces
- ☛ $e_3 = [0 \ \ 0 \ \ 1]^{\mathrm{T}} \in \mathbb{R}^3$

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$$\tau \in \mathbb{R}^3, \; \mathcal{K} = \begin{bmatrix} -k_f & -k_f & -k_f & -k_f \\ 0 & -k_f D & 0 & k_f D \\ k_f D & 0 & -k_f D & 0 \\ -k_\tau & k_\tau & -k_\tau & k_\tau \end{bmatrix} \in \mathbb{R}^{4\times 4}$$

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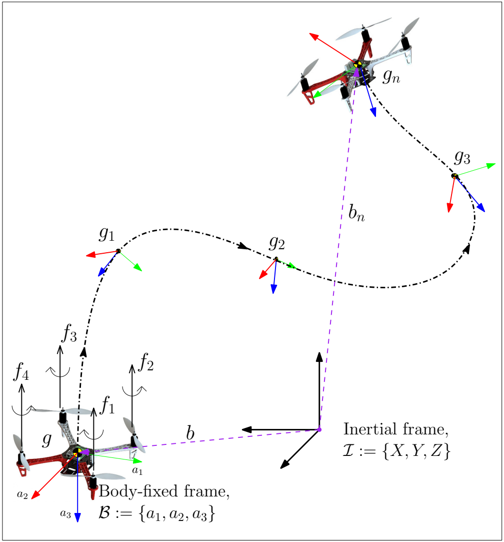
Integrated trajectory generation and control



Integrated trajectory generation and control

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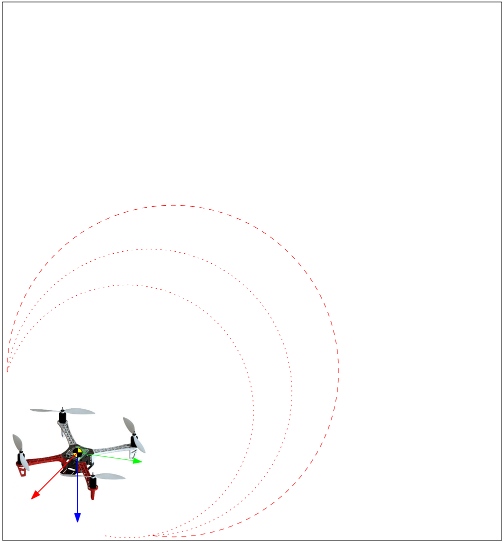

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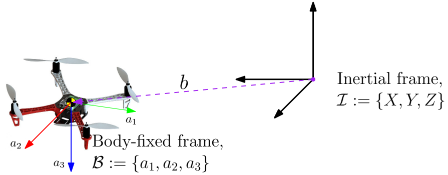

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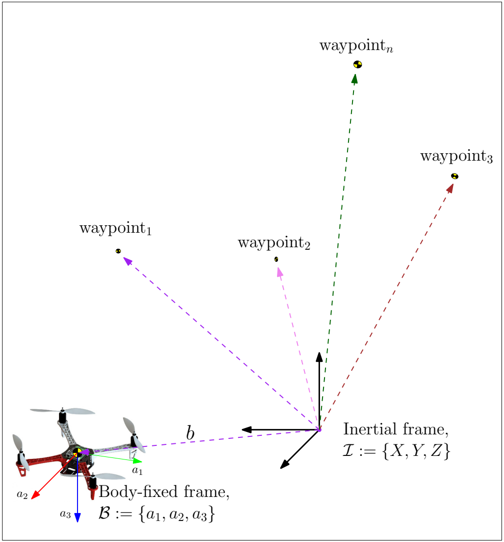

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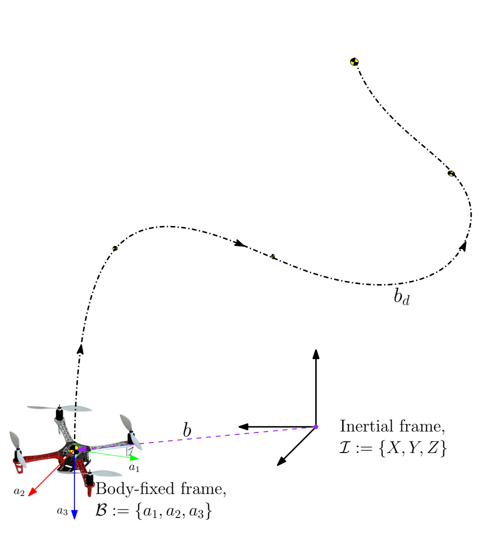

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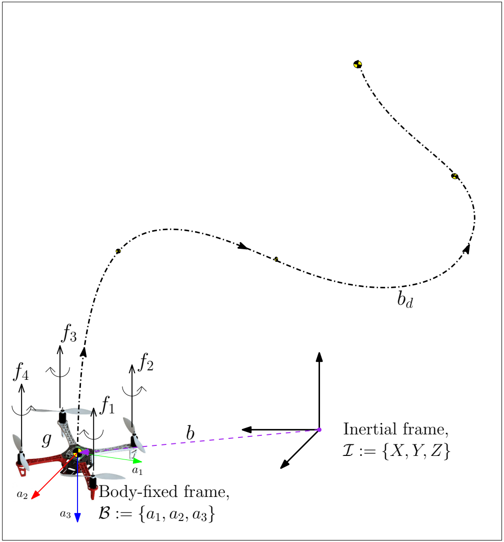
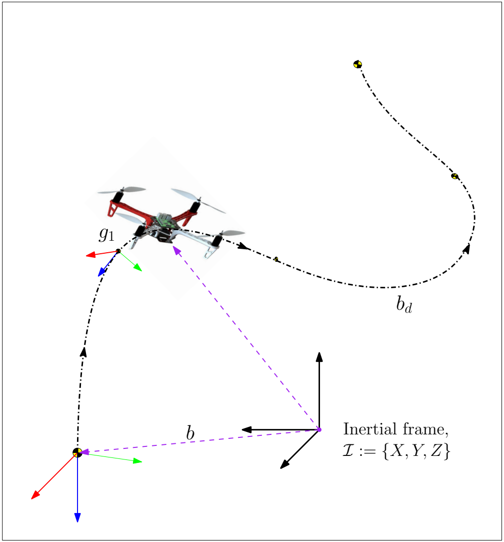
  - Compute control thrust f
  - Generate the desired attitude trajectory, $R_d(t)$
  - Compute the attitude control torque



Integrated trajectory generation and control

- Off-line
  - Inspect the environment
  - Initialize the UAV
  - Select desired waypoints in $\mathbb{R}^3$ w.r.t $\mathcal{I}$
  - Generate a desired trajectory $b_d(t) = \mathrm{C}^2(\mathbb{R}^3)$
- On-line
  - Compute control thrust f
  - Generate the desired attitude trajectory, $R_d(t)$
  - Compute the attitude control torque
- UAV tracking trajectory



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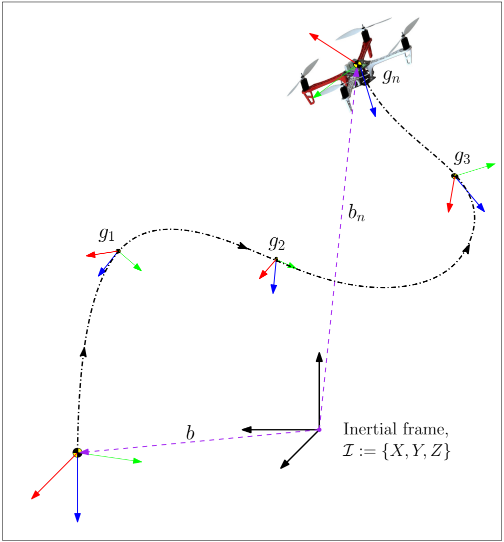


Tracking errors definition

- Position tracking error: $\tilde{b} := b - b_d$
- Translational velocity tracking error: $\tilde{v} := v - v_d = \dot{\tilde{b}}$
- Attitude tracking error: $Q = R_d^{\mathrm{T}} R$
- Angular velocity tracking error: $\omega = \Omega - Q^{\mathrm{T}} \Omega_d$
- Translational error dynamics in inertial frame $\mathcal{I}$ is:

$$\dot{\tilde{b}} = \tilde{v} = v - v_d, \quad m\dot{\tilde{v}} = mge_3 - fr_3 - m\dot{v}_d, \tag{4}$$

- True attitude of the body is represented by $r_3 = Re_3$
- Control force vector, $fr_3 = \bar{\varphi}_c$ is expressed in $\mathcal{I}$

Translational Control Law

Theorem

Consider the translational dynamics given in (4); define

$$z(t) = \frac{\tilde{b}}{\left(\tilde{b}^T \tilde{b}\right)^{1-\frac{1}{p}}}, \tag{5}$$

where $p \in (1, 2)$. The feedback control $u \in \mathbb{R}^3$ given by,

$$u = ge_3 - \dot{v}_d + k\dot{z} + k\tilde{b} + \frac{kP(\tilde{v} + kz)}{[(\tilde{v} + kz)^T P(\tilde{v} + kz)]^{1-\frac{1}{p}}}, \tag{6}$$

where control gain matrix $P \succ 0$, stabilizes the translational error dynamics

$$\dot{\tilde{v}} = ge_3 - \dot{v}_d - u; \text{ where } mu(t) = f(t)r_3,$$

in finite time.

Generating desired attitude trajectory

- Attitude is controlled to achieve stable tracking of the desired translational motion
- Desired attitude $R_d = [r_{2d} \times r_{3d} \quad r_{2d} \quad r_{3d}] \in \mathrm{SO}(3)$ is generated as follows:

$$r_{3d} = R_d e_3 = \frac{u}{\|u\|}. \tag{7}$$

- Now compute $r_{2d} = \frac{r_{3d} \times s_d}{\|r_{3d} \times s_d\|} = R_d e_2$, by selecting an appropriate $s_d(t) \in \mathcal{C}^2(\mathbb{R}^3)$ such that it is transverse to r_{3d}

Theorem

Let $r_{3d} = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}^{\mathrm{T}} \in \mathbb{S}^2 \subset \mathbb{R}^3$ be a known unit vector as given in (7). The vector

$$s_d = \begin{bmatrix} a_2 + a_3 & a_3 - a_1 & -a_1 - a_2 \end{bmatrix}^{\mathrm{T}} \tag{8}$$

is orthogonal to r_{3d}.

Finite-time Stable Attitude Tracking Control on $\mathrm{TSO}(3)$

Theorem

The feedback control law τ_c for attitude control is given by

$$\begin{aligned}\tau_c = {} & J\left(Q^{\mathrm{T}}\dot{\Omega}_d - \frac{\kappa H(s_K(Q))}{\left(s_K^{\mathrm{T}}(Q)s_K(Q)\right)^{1-1/p}}w(Q,\omega)\right)\\ & + (Q^{\mathrm{T}}\Omega_d)^{\times}J\left(Q^{\mathrm{T}}\Omega_d - \kappa z_K(Q)\right) + \kappa J\left(z_K(Q)\times Q^{\mathrm{T}}\Omega_d\right)\\ & + \kappa J(\omega + Q^{\mathrm{T}}\Omega_d)\times z_K(Q) - k_p s_K(Q)\\ & - \frac{L\Psi(Q,\omega)}{\left(\Psi(Q,\omega)^{\mathrm{T}}L\Psi(Q,\omega)\right)^{1-1/p}},\end{aligned} \tag{9}$$

where

$$\Psi(Q,\omega) = \omega + \kappa z_K(Q), \text{ and } H(x) = I - \frac{2(1-1/p)}{x^{\mathrm{T}}x}xx^{\mathrm{T}}. \tag{10}$$

Then the feedback attitude tracking error dynamics is stabilized to $(Q,\omega) = (I,0)$ in finite time.

Stability of the Overall Feedback System on $\mathrm{TSE}(3)$

Theorem

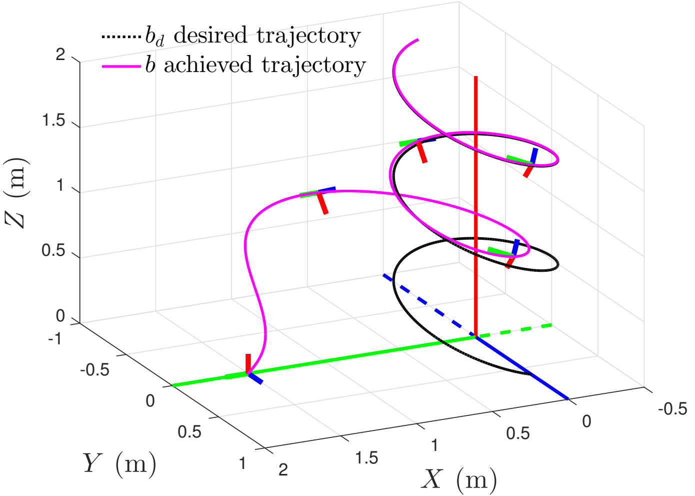
The overall feedback control system given by the tracking error kinematics and dynamics is finite-time stable for the generated state trajectory $(b_d(t), R_d(t), v_d(t), \Omega_d(t)) \subset \mathrm{TSE}(3)$. Moreover, the domain of convergence is almost global over the state space.

Lyapunov functions used (for translational and rotational motions):

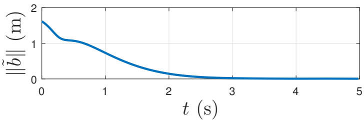
$$V_{tr}(\tilde{b}, \tilde{v}) = \frac{1}{2}\left(k\tilde{b}^{\mathrm{T}}\tilde{b} + (\tilde{v} + kz)^{\mathrm{T}}(\tilde{v} + kz)\right), \tag{11}$$

$$V_{rot}(Q, \omega) = k_p\langle K, I - Q\rangle + \Psi(Q, \omega)^{\mathrm{T}} J \Psi(Q, \omega). \tag{12}$$

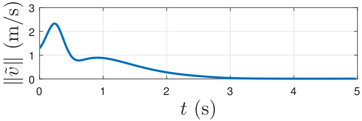
Numerical Results



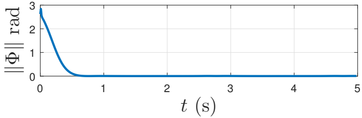
Numerical Results



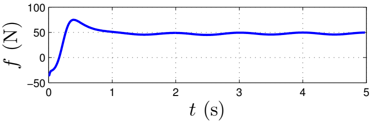
(a) Norm of position tracking error



(b) Norm of velocity tracking error



(c) Attitude tracking error



(d) Magnitude of total thrust

Conclusion

- Position trajectory tracking on $\mathbb{R}^3$ through given waypoints
- Thrust force control for finite-time stable position trajectory tracking
- Attitude trajectory generation on $\mathrm{SO}(3)$ based on desired thrust direction
- Finite time stable attitude control to track required attitude
- Validation of integrated guidance and control

Future/Ongoing Work

- Optimal C^2 position trajectory generation with known bounds on velocities and accelerations
- Optimal C^2 position and pointing trajectory generation through given waypoints (on $\mathbb{R}^3 \times \mathbb{S}^2$)
- Experiments to be carried out in Autonomous Unmanned Systems Laboratory (AUSL), Syracuse Center of Excellence

THANKS

Questions and Comments