SC 631: Games and Information	Autumn 2014

Lecture 13: September 19

Instructor: Ankur A. Kulkarni *Scribes: Siddhant, Kshitij, Mridul, Dev*

Note: *LaTeX template courtesy of UC Berkeley EECS dept.*

13.1 Introduction

It was suggested that Nash equilibrium represents *self-enforcing agreements*. That is, if we allow the players to have a pre play discussion and they arrive to some Nash Equilibrium, then they are compelled to play the strategy enforced by that equilibrium.
In this lecture, we argue that this may not be the case. We present some examples and then discuss the situation before and after an agreement is made.

13.2 Preliminaries

(x, y) denotes a strategy profile where $P1$ plays x and $P2$ plays y. $U^i(x, y)$ represents ith player's payoff in a 2 player game on the strategy profile (x, y).

Pareto-dominant Nash equilibrium Suppose (x, y) is a Nash euilibrium, then it is Pareto-dominant if for every other equilibrium $(\overline{x}, \overline{y})$

$$U^i(x, y) \geq U^i(\overline{x}, \overline{y}),\ \forall i$$

and at least one of the inequalities is strict.

Risk-dominant Nash equilibrium Suppose (x_1, x_2) and (y_1, y_2) are Nash equilibria, then (x_1, x_2) risk dominates (y_1, y_2)

if $(U^1(x_1, x_2) - U^1(y_1, x_2))(U^2(x_1, x_2) - U^2(x_1, y_2)) > (U^1(y_1, y_2) - U^1(x_1, y_2))(U^2(y_1, y_2) - U^2(y_1, x_2))$

13.3 Examples

We present three games and discuss their outcomes.

13.3.1 Game 1

		Bob	
		c	d
Alice	c	(9,9)*	(0,8)
	d	(8,0)	(7,7)*

(c, c) is Pareto-dominant as the players'payoff is more than the payoff of any other strategy profile.
Also, (d, d) is Risk-dominant Nash equilibrium which can be verified by its definition.

$$(U^{Alice}(d,d) - U^{Alice}(c,d))(U^{Bob}(d,d) - U^{Bob}(d,c)) > (U^{Alice}(c,c) - U^{Alice}(d,c))(U^{Bob}(c,c) - U^{Bob}(c,d))$$
$$(7-0)(7-0) > (9-8)(9-8) \implies 49 > 1 \text{ which is true.}$$

1. **The game is noncooperative :** Since the players cannot communicate, Alice is not certain that Bob will play c. She, therefore, might choose to play d, which gives her a payoff of at least 7, whereas with c she may get nothing. She may also reason out that Bob can make the same reasoning which makes Alice more likely to place c. The same logic works for Bob. We also see that (d,d) is safer (risk dominant). With all these given, we do not assert that they must play d. But playing d is not unreasonable.

2. **Allowing preplay communication:** Now suppose pre-play communication is permitted and both players agree to play (c,c). We then ask two questions here: (c,c) being equilibrium, is the agreement enforceable? Does it remove each players doubt about the other one playing c? Suppose Alice is careful and she gives another thought before choosing c.
   She thinks the following :

   If Bob does not trust me and plays d inspite of the agreement, he would still want me to play c which gives him 8 rather than 7. Also if he plays c, he still wants me to play c. So the agreement is not binding him. Bob might want me to play c, which I already knew because it always gave hime more payoff. It conveys no information about his play. Since, he can reason in the same way about me, none of us gets any information from the agreement about what other plays. So the agreement does not change the incentives which obtain in the absence of it. So, I will choose d.

   The main point of this example is not that Alice will play d. But rather that *Bob agreeing to play c has not provided Alice any information that Bob will play c*. The agreement does not signal that Bob wants to play c (since it is not binding), but rather that Bob wants Alice to play c. But Bob wants Alice to play c regardless of whether he plans to play c – since in that case he gets 9 (c) and 8 (d) which is better than 0 and 7.

   The agreement puts following things:

   (a) Since it is not binding , Bob may not play c.

   (b) But since he agreed for it, he wants alice to play c.

   The question arises here, why would Bob deviate from c if he gets a payoff of 9? Well, we can reason in many different ways about what a player thinks. Maybe he also does not trust Alice to play c.

   Remember, common knowledge of rationality does not lead us to choose (9,9). In fact, it does not let us go any further from the given matrix. It is as if we started from a larger matrix and arrived at it where no strategy dominates other.

13.3.2 Game 2 (Battle of the sexes)

Alice \ Bob	b	f
b	(2,1)*	(0,0)
f	(0,0)	(1,2)*

(b,b) and (f,f) are Nash equilibrium. But there is no Pareto-dominant equilibrium as there is there is no strtegy profile which give the highest payoff and no Risk-dominant equilibrium because

$(U^{Alice}(b,b) - U^{Alice}(f,b))(U^{Bob}(b,b) - U^{Bob}(b,f)) = (U^{Alice}(f,f) - U^{Alice}(b,f))(U^{Bob}(f,f) - U^{Bob}(f,b)$ which does not satisfy its defintion.

1. **The game is noncooperative :** There are two nash equilibrium in this game but neither player can decide to choose between b(ballet) or f(fight) hence without a pre-play communication they have no clear motivation of choosing between these two strategies.

2. **Allowing preplay communication:** When a pre-play communication is made and both agree to play either (b,b) or (f,f) then this agreement is self-enforcing as we can reason for it as follows :
Its not that Alice takes this agreement as direct signal that Bob will play this agreed strategy but like previous example she knows that Bob has agreed on this strategy because otherwise Bob's payoff decreases. He wants her to keep this agreement but unlike previous example here she knows Bob is himself keen on keeping the agreement hence this agreement is self-enforcing.

Thus, in this game Nash equilibrim is indeed self-enforcing.

13.3.3 Game 3

		Bob	
		c	d
Alice	c	(100,100)*	(0,8)
	d	(8,0)	(7,7)*

This game is similar to Game 1. Here, (c,c) is Pareto-dominant as well as Risk-dominant equilibirum.

The same reason in Game 1 works here as changing the payoff is adding extra information. Though the chances of playing (c,c) is now high but pre-play communication won't improve its chances further. Hence, the agreement is still not self-enforcing.

13.4 Conclusion

In a noncooperative game, the players play their strategy based on some incentives available to them whereas in a cooperative game, an agreement is useful only when it changes the incentives that had been in its noncooperative version.
There are two ways to change the incentives. Either the payoffs are changed or the information is changed. In our case, payoffs remain the same. Therefore, agreement is useful only when it changes the players information; specifically their information about how others will play.
We can also see this in terms of observations and state of the world. Information about an event E is acquired by observing a parameter that depends on whether or not E obtains.
In the first game, Alice knows that if she plays c, it will be beneficial for Bob. After the agreement, the situation remains the same. Therefore, Alice cannot calculate the actual state of the world.

References

[] Robert Aumann , Nash Equilibria Are Not Self-Enforcing *Chapter 34, Economic Decision Making: Games, Econometrics, and Optimisation: Essays in Honor of Jacques Dreze*, 1990.