SC 631: Games and Information **Autumn 2014**

Lecture 15: September 26

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15.1 Recap

In the previous lecture, we discussed dynamic games and how to represent them as trees. In this lecture, we will look at a generalization of dynamic games where players have incomplete information, i.e. they do not know their exact location in the tree.

15.2 An Example

Consider a game where P_1 plays, followed by P_2. P_1 has 3 strategies—L, M, and R. P_2 has 2 strategies—L and R. This game can be represented as a tree in the following way.

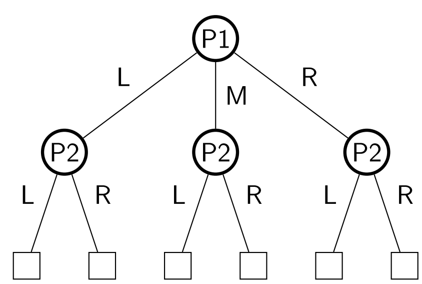

Figure 15.1: Dynamic game

Now suppose that P_2 knows whether P_1 has played L or not, but cannot distinguish between M and R. This can be represented as the following tree, where at any point in the game, a player can distinguish between nodes of different color but not between nodes of the same color.

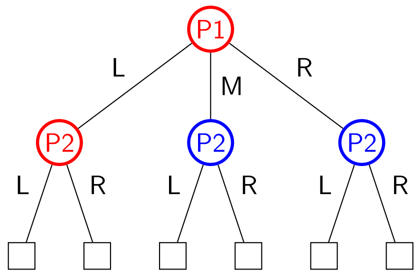

Figure 15.2: Dynamic game with incomplete information

Strategies of P_2 are a function of L and L^C. Since P_2 cannot distinguish between M and R, his strategies are same for both cases. The following is the set of possible strategies for P_2—

$$\gamma_1^2 = \begin{cases} L & \text{if } P_1 \text{ plays L} \\ L & \text{if } P_1 \text{ plays M or R} \end{cases}$$

$$\gamma_2^2 = \begin{cases} R & \text{if } P_1 \text{ plays L} \\ R & \text{if } P_1 \text{ plays M or R} \end{cases}$$

$$\gamma_3^2 = \begin{cases} L & \text{if } P_1 \text{ plays L} \\ R & \text{if } P_1 \text{ plays M or R} \end{cases}$$

$$\gamma_4^2 = \begin{cases} R & \text{if } P_1 \text{ plays L} \\ L & \text{if } P_1 \text{ plays M or R} \end{cases}$$

The set of possible strategies for P1 is L, M, R.

The most general setup for such games is called an **extensive-form dynamic game**, defined in the next section.

15.3 Extensive-form dynamic games

15.3.1 Definition

An extensive-form dynamic game for N players is a tree with the following properties:

- A specific vertex indicating the **starting point**
- A **payoff** for each player at each terminal node
- A partition of the nodes of the tree into N **player sets**
- A subpartition of each player set into **information sets** $\{\eta_j^i\}$ such that *the same number of branches emanate from each node belonging to the same information set* and *no node follows another node in the same information set*

15.3.2 Comments

- Since the graph is a tree and does not contain any loops, there is an unique path from the root to any leaf node. In other words, "all histories are distinct".
- Leaves may or may not be included in the set of nodes to be partitioned, as no actions arise from them hence they make no difference.
- Players do not usually collect payoffs as they go. Payoffs are given after reaching the leaf node.
- When we say that no node follows another node in the information set, we mean that the player simply knows that he has played. He need not know what he has played as there are ways to forget that information. Hence, the model is more like a 'model of awareness'.

15.3.3 An Example

In the following game, nodes that are marked with the same player number belong to the same player set. Nodes in the same player set but in different information sets are distinguished by color.

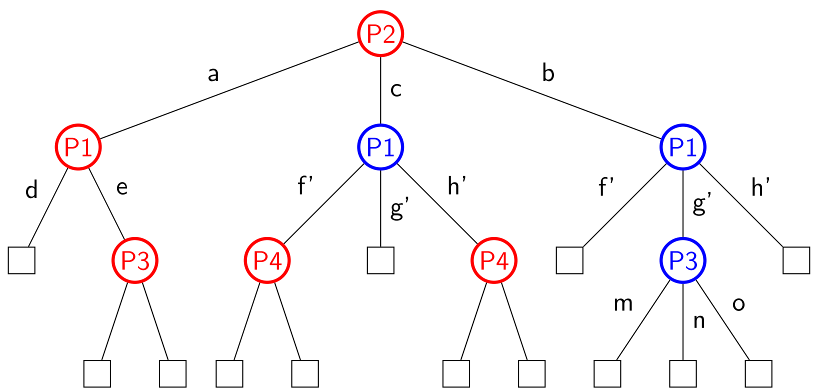

Figure 15.3: An example of an extensive-form dynamic game

Note that if the root node had been in P_1's player set, and P_1 played either c or b on his first turn, he would end up in the same information set. Therefore, he would know that he had played, but not what he had played. Although this may seem strange, this has practical use because in some real-world systems, the "players" may have finite memory.

15.4 Strategies

As we can guess by looking at the example in section 15.2, strategies are maps from information sets to actions.

15.4.1 Definition

Denote by I^i the set of all information sets of P_i. For any $\eta^i \in I^i$, let $U^i_{\eta^i}$ be the set of actions available to P_i at η^i. A strategy for P_i is a function $\gamma^i : I^i \to u^i$, where $u^i = \bigcup_{\eta^i \in I^i} u^i_{\eta^i}$ such that

$$\gamma^i(\eta_i) \in u^i_{\eta^i} \quad \forall \eta^i \in I^i$$

15.5 Special kinds of games

Since the extensive-form dynamic game is a more general definition than the ones we have encountered before, we can try to see what properties those games have in this definition.

- In a **simultaneous move** or **static** game, no player has knowledge of the other's strategy. Therefore, *each player has only one information set.*
- At the other extreme is the **perfect information** game, where each player knows the sequence of strategies played at any point. In other words, *each node is in a different information set*, or each information set is a singleton.
- In a **single act** game, each player can play atmost once. Therefore, *each path starting from the root node intersects the player set of each player atmost once.* This means that all static games are also single act games, because each player has only one information set, and no two nodes on the same path can be in one information set by definition.

15.6 Comparing games

It is possible to "compare" two games that have the same tree, if all players know more about their location in one game than in the other. This can be formalized in the following definition.

15.6.1 Definition

A game (I) is **informationally inferior** to a game (II) if $\forall$ players i and $\forall$ information sets $\eta^i_{j,II}$ $\exists$ $\eta^i_{k,I}$ such that $\eta^i_{j,II} \subseteq \eta^i_{k,I}$ and atleast one inclusion is strict.