**SC 631: Games and Information** **Autumn 2014**

# Lecture 1: October 10

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**Note**: *LaTeX template courtesy of UC Berkeley EECS dept.*



This lecture's notes illustrate some uses of various LaTeX macros. Take a look at this and imitate.

**Definition**
In an extensive form of a single act nonzero sum finite game with a fixed order of play, a player Pi is said to be a precedent of another player Pj if the former is situated closer to the vertex of the tree than the later. And the extensive form is called nested if each player has the information which is available to its precedents.
**Single Act Games.**

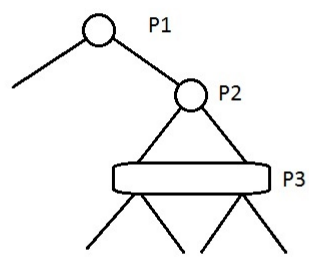


Figure 1.1:

This is nested because P2 knows what P1 knows and P3 knows what P2 and P1 knows.

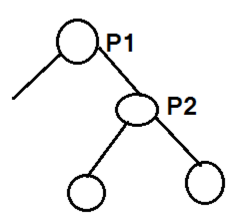


Figure 1.2:

This is also nested due to similar reasoning.
It is ladder nested if the only difference between the info of $P_i$ and $P_{i-1}$(his immediate precedent) is that corresponding to the actions of Pi-1 and only at nodes corresponding to branches emerge from singleton info

sets of $P_{i-1}$.

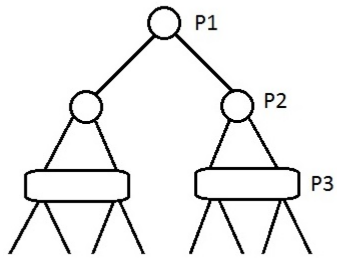


Figure 1.3:

This is nested and also ladder nested . This is because P2 knows what P1 has played. The information which P3 has is equal to that of P2. Hence it is ladder nested.

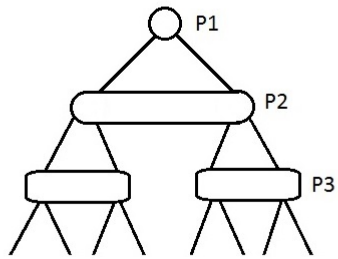


Figure 1.4:

In this case the amount of information which P3 has is more than that which P2 has. This information is about P1. Thus it is nested but not ladder nested..

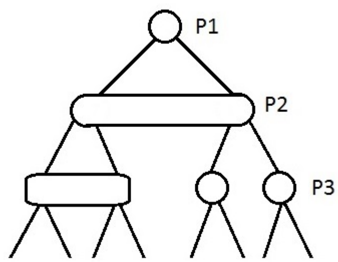


Figure 1.5:

P3 knows some additional information about actions of P2. But that is not his only additional information. This case like the previous one is nested, but not ladder nested.

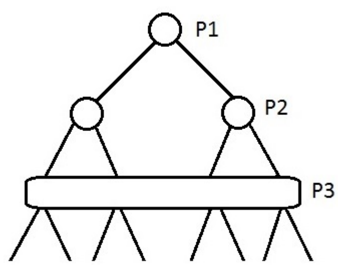


Figure 1.6:

This is also a case which is not nested..

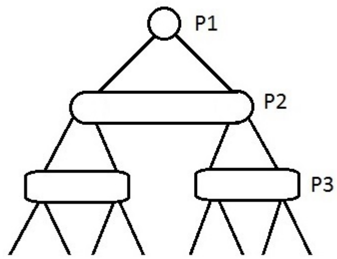


Figure 1.7:

Equivalent to previous cases far as information is concerned. In the first case P2 doesnt know about actions of P3 which is as good as P3 acting first and P2 acting subsequently.

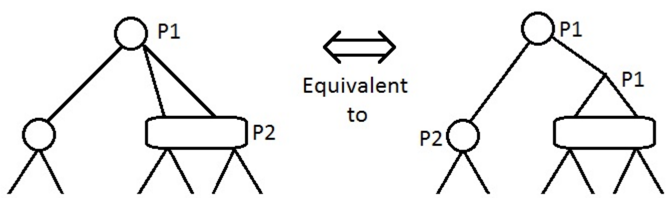


Figure 1.8:

**Defenition**
Consider a singleton information set of Pi and the subtree of all branches that enter into a particular info set of $P_{i+1}$(immediate follower) and their children. This is called a subextensive form.
**Algorithm** for finding the equivalent of ladder nested:

- For each info set of last acting player, determine the subextensive form that includes all players that includes all players that have the same info as this info set.
- Solve this static game.(in pure stratergies)
- Replace the subextensive form with a branch of equivalent action of the first acting of player in subextensive form.
- Repeat until you are left with only branches of first acting player at root node.
- If there are multiple equivalent repeat the procedure for each.

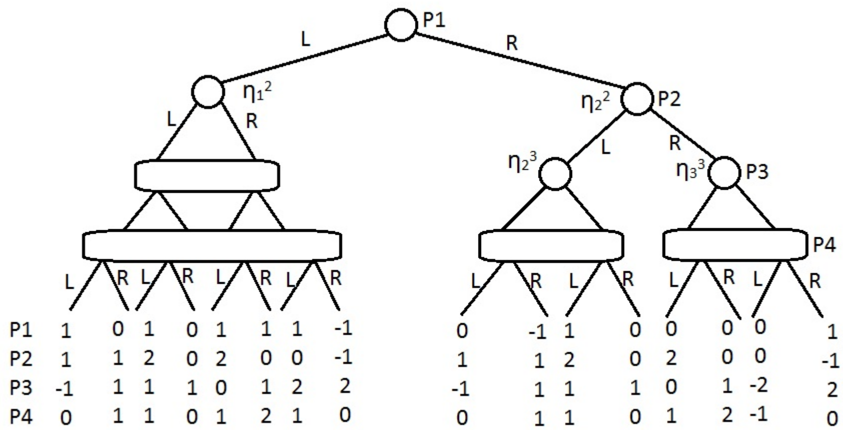


Figure 1.9:

Start from of the last acting player . Consider the subextensive form in which everyone has the same info as last player.

**Left Game**

$U^{2^*} = L, U^{3^*} = L, U^{4^*} = L$

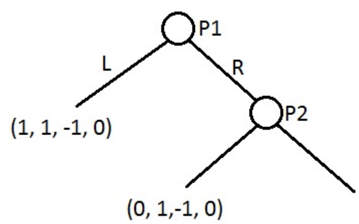


Figure 1.10:

Right game 1; $U^{3^*} = L, U^{4^*} = L$
Left game 1;$U^{3^*} = R, U^{4^*} = L$
Feature of equillibria, Equillibria of static game will not come up with the above pattern. By this algorithm we are finding a nash equilibrium. But there might be other Nash equilibriums also. Equillibria generated by this algorithm are equivalent of the delayed commitment type (where action are committed only after seeing info) .

$$\gamma^{2^*}(\eta) = \begin{cases} L\eta = \eta_1^2 \\ R\eta = \eta_2^2 \end{cases}$$

$$\gamma^{2^*}(\eta) = \begin{cases} L\eta = \eta_1^2 \\ L\eta = \eta_2^2 \\ R\eta = \eta_3^2 \end{cases}$$

**Definition**
For an N person single act game(I), let $J$ denote the set of games that are informationally inferior to (I). Let $(\gamma^{1*}, \gamma^{2*}, \gamma^{3*}.........\gamma^{N*})$ be a Nash equilibrium of (I) and let $J^*$ be the number of games in $J$ of which then Nash Equilibrium is a Nash Equilibrium. Then $(\gamma^{1*}, \gamma^{2*}, \gamma^{3*}.........\gamma^{N*})$is a NE of delayed commitment type if there exists no other Nash Equilibrium of (I) which is an equivalent of fewer than $J^*$ games in $J$.
$\Rightarrow$ Even if a game is not ladder nested one can talk of delayed commitment eq.

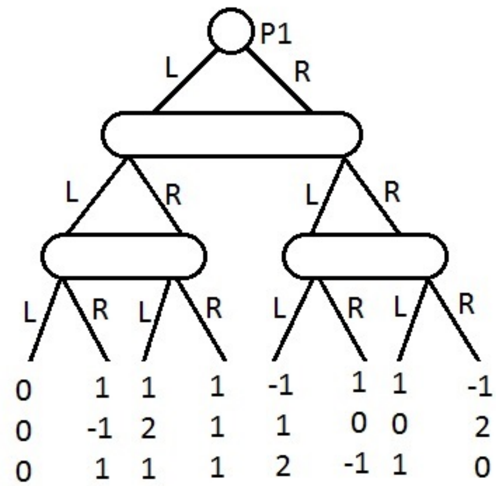


Figure 1.11:

$$\gamma^{1*} = L$$

$$\gamma^{2*} = L$$

$$\gamma^*(\eta) = \begin{cases} L & \text{if} \quad u' = L \\ R & \text{if} \quad u' = R \end{cases}$$

$$\gamma^{1*} = R$$

$$\gamma^{2*} = L$$

$$\gamma^{3*} = R$$