SC 631: Games and Information **Autumn 2014**

Lecture 19: October 14

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This lecture's notes illustrate some uses of various LaTeX macros. Take a look at this and imitate.

19.1 Multistage Games with Observed Actions (Feedback Games)

We will consider a particular class of Multi-act games, called *feedback games*.

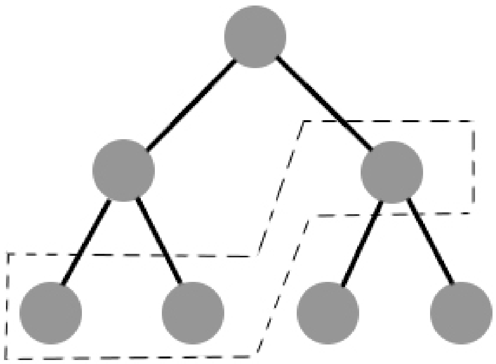
Definition 19.1.1 ***A feedback game in extensive form is :***

- *A tree divided into 'stages'*
- *First acting player at each stage has singleton information set*
- *Information sets of other players are such that none of them includes nodes corresponding to branches emanating from 2 or more information sets of the first acting player*
- *No information set contains nodes from more than 1 stage*
- *Payoffs are stored at leaf nodes*

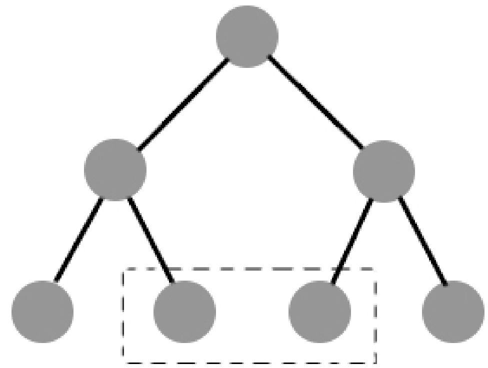
Further, we can assume that :

- *Each stage is a single-act game*
- *If single-act games at each stage are ladder nested(or nested) then it is a ladder nested(or nested) game*
- *Every player plays once in each stage*
- *Each player knows the state of the game at every level of play*

Therefore such trees are not possible :



Set contains nodes of the tree belonging to different levels of play



Set contains nodes corresponding to branches emanating from 2 different information sets of the other player

19.2 Nash Equilibrium in feedback games

Let γ^i be the strategy for player P_i, such that

$$\gamma^i = \gamma_1^i, \gamma_2^i, ...\gamma_k^i \tag{19.1}$$

where k is total no of stages and γ_j^i is the strategy of P_i at stage j.

Then Nash Equilibrium for the game will be,

$$J^i(\gamma^{i*}, \gamma^{-i*}) \leq J^i(\gamma^i, \gamma^{-i*}) \qquad \forall\, \gamma^i \tag{19.2}$$

19.2.1 Feedback Nash Equilibrium

Let γ^{1*}, γ^{2*},..., γ^{N*} be such that they satisfy Nash Equilibruim condition at every stage, then such a Nash Equilibrium is called **Feedback Nash Equilibruim**.
A feedback N.E can be found using backward induction.

N.E condition at stage k,

$$J^i(\gamma_1, ...\gamma_{k-1}, \gamma_k^{i*}, \gamma_k^{-i*}) \leq J^i(\gamma_1, ...\gamma_{k-1}, \gamma^i, \gamma^{-i*}) \qquad \forall\, \gamma_k^i,\ \forall\, i,\ \forall\, \gamma_1, ...\gamma_{k-1} \tag{19.3}$$

where $\gamma_t = \gamma_t^1$,...γ_t^N, i.e. strategy of all N players at stage t.

Therefore, using backward induction, N.E condition at stage k-1 would be,

$$J^i(\gamma_1, ...\gamma_{k-2}, \gamma_{k-1}^{i*}, \gamma_{k-1}^{-i*}, \gamma_k^*) \leq J^i(\gamma_1, ...\gamma_{k-2}, \gamma_{k-1}^{i}, \gamma_{k-1}^{-i*}, \gamma_k^*) \qquad \forall\, \gamma_{k-1}^i,\ \forall i,\ \forall \gamma_1, ...\gamma_{k-2} \tag{19.4}$$

Proceeding in this manner, N.E condition at stage 1 would be,

$$J^i(\gamma_1^{i*}, \gamma_1^{-i*}, \gamma_2^*, ...\gamma_k^*) \leq J^i(\gamma^i, \gamma_1^{-i*}, \gamma_2^*, ...\gamma_k^*) \qquad \forall\, \gamma_1^i,\ \forall i, \tag{19.5}$$

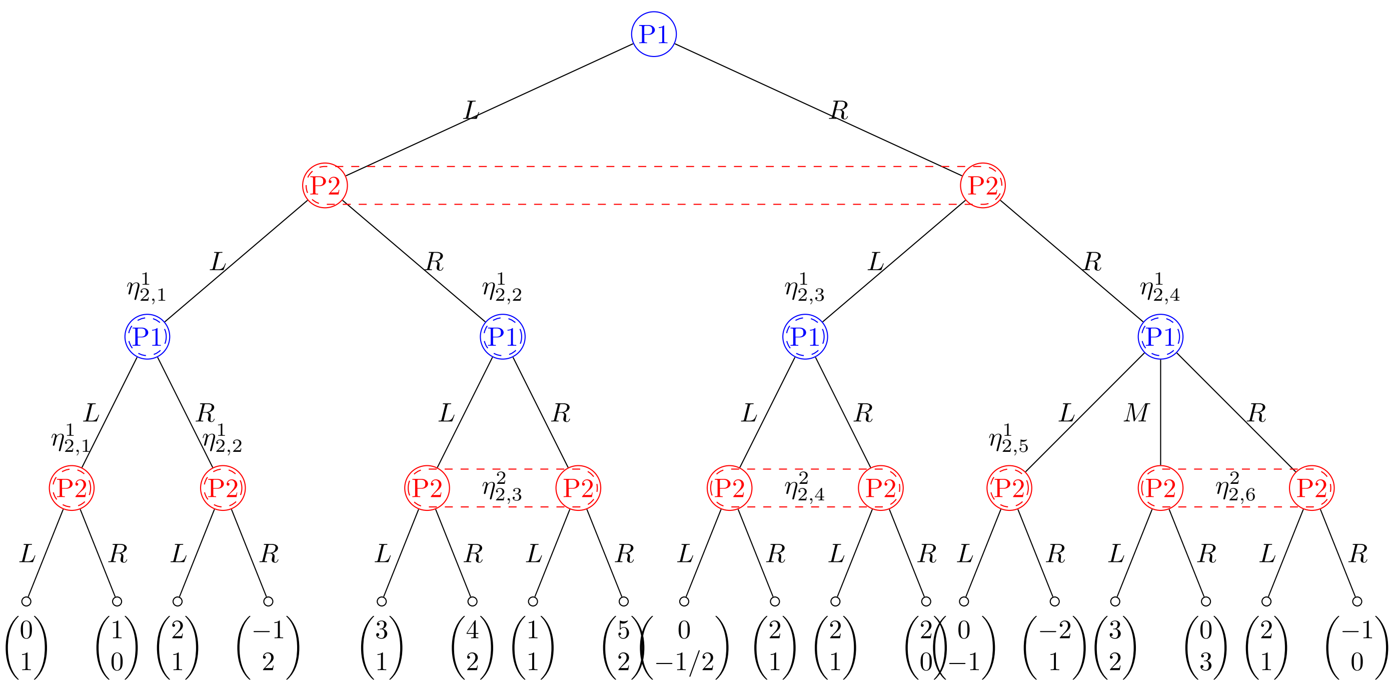
Theorem 19.2.1 *Every Feedback NE is a NE*
Hint: Proof by Contradiction

19.3 Algorithm for Feedback Nash Equilibrium

- Starting from stage k solve each single-act game for information set of first acting player.
- Delete subtree and replace with equilibrium payoff Work this way to the root of tree.
- If single-act game admit mulitple equilibrium, repeat for each.

19.3.1 Example

Consider the following game tree in extensive form.



Lemma 19.1 *A Feedback Nash Equilibrium is of type Delayed Commitment Type if it induces a delayed commitment equilibrium in each stage.*

It can be shown that the game has 4 single-act games each starting from P1 node at level 2 in the tree. Equilibrium strategies of these 4 single-act games are as follows:

First single-act game:

$$\gamma_2^{1*}\left(\eta_{2,1}^1\right) = L \tag{19.6}$$

$$\gamma_2^{2*}\left(\eta_{2,1}^2\right) = \begin{cases} R & \text{if } U_2^1 = L \\ L & \text{Otherwise} \end{cases} \tag{19.7}$$

Second single-act game:

$$\gamma_2^{1*}\left(\eta_{2,2}^1\right) = R \tag{19.8}$$

$$\gamma_2^{2*}\left(\eta_{2,3}^2\right) = L \tag{19.9}$$

Third single-act game:

$$\gamma_2^{1*}\left(\eta_{2,3}^1\right) = L \tag{19.10}$$

$$\gamma_2^{2*}\left(\eta_{2,4}^2\right) = L \tag{19.11}$$

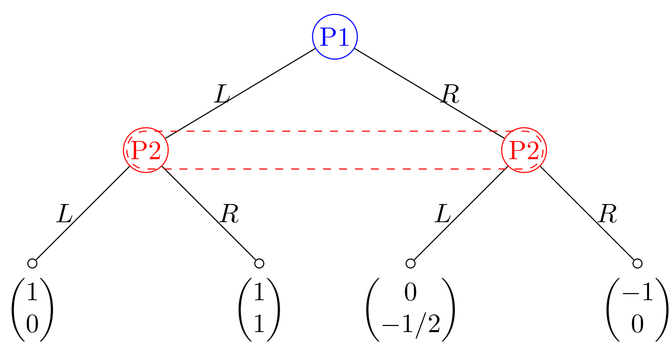
Fourth single-act game: This has two equilibria. One from the left subtree and second from the right sub tree. From the right sub-tree we get static equlibrium where as from left sub-tree we get equilibrium of delayed commitment type. Let us analyze them one by one.

Case1: Static equilibrium

$$\gamma_2^{1*}\left(\eta_{2,4}^1\right) = R \tag{19.12}$$

$$\gamma_2^{2*}\left(\eta_{2,4}^2\right) = L \tag{19.13}$$

Now following step 2 of the algorithm mentioned above we can reduce the tree to the following subtree:



The equilibrium of the game is:

$$\gamma_1^{1*} = R \tag{19.14}$$

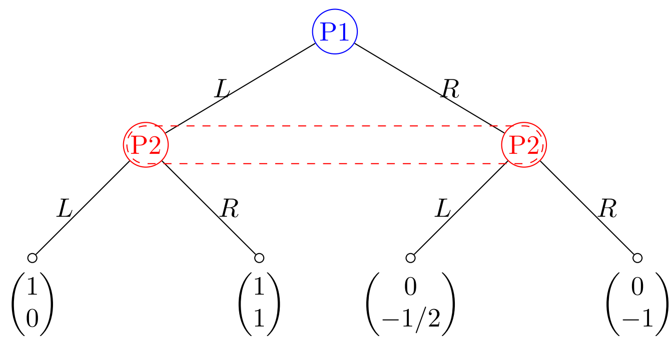
$$\gamma_1^{2*} = L \tag{19.15}$$

Case2: Delayed Commitment Type Equilibrium

$$\gamma_2^{1*}\left(\eta_{2,4}^1\right) = L \tag{19.16}$$

$$\gamma_2^{2*}\left(\eta_{2,5}^2\right) = \begin{cases} L & \text{if } U_2^1 = L \\ R & \text{Otherwise} \end{cases} \tag{19.17}$$

Now following step 2 of the algorithm mentioned above we can reduce the tree to the following subtree:



The equilibrium of the game is:

$$\gamma_1^{1*} = R \tag{19.18}$$

$$\gamma_1^{2*} = R \tag{19.19}$$

Note: This is a Feedback Equilibrium of Delayed Commitment Type.