Consensus based Deviated Cyclic Pursuit for Target Tracking Applications

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***Abstract*— Decentralised strategies for target tracking has been studied extensively in the literature. In this paper, we present a strategy which gives the flexibility to change formation while the UAVs are executing a mission. A target is assumed to be enclosed if the UAVs encircle the target at a given radius while being uniformly distributed around it. If we want to close on to the target or if we want to enclose a wider area around the target, the radius needs to change. In a decentralised scheme, all the vehicles have to be informed about the new radius, which is difficult when the number of vehicles are larger. The strategy proposed in this paper can induce a change in radius by changing a parameter in one of the vehicles picked randomly from the group. The idea is based on consensus in deviated pursuit. The control law is simple to compute and requires only bearing angle measurement. We have analysed the system assuming unicycle kinematics. The strategy is then implemented in 6-DOF dynamical model of the UAVs and simulated in MATLAB and in Hardware-in-loop simulator.**

I. INTRODUCTION

Several applications such as convoy protection, natural resource monitoring, geographical exploration, etc. require persistent monitoring of a point of interest from multiple directions. A cooperating team of multiple autonomous vehicles can be used for such tasks as they offer several advantages such as reliability, robustness, scalability and better efficiency. In this paper we propose a control strategy for multiple autonomous vehicles which makes them move from any initial position towards a specific target, and upon reaching the target they continue to move around it with uniform distribution. Moreover, the distance to the target can be easily controlled in a decentralised manner.

Most target tracking strategies discussed in the literature ([1] - [10] to list a few) rely on the position information either absolute or relative. Absolute position is defined in a global reference frame whereas the relative position information can be defined in local body frame of individual agent. Vision sensor can be used to get the relative position information using range and bearing angle measurements. Both these quantities are defined in the local frame. Vision based formation control strategies are addressed in [11] - [15] . Moshtagh et al. [11] have proposed a vision based control law to achieve a circular formation which only requires bearing angle information of the neighbors. Every agent is assumed to be equipped with a vision sensor for measuring the bearing angle which is a quantity defined in the local body frame of the agent. The agents finally converge to a circular formation but the center of the formation cannot be predicted a priory. For target enclosing applications it is desired that the formation finally converges about a specific point of interest (target). Ground vehicle tracking by multiple UAVs utilising vision input is discussed in [16] where the target tracking control and coordination control are treated separately. Here the tracking control is a function of both the bearing angle and range measurements whereas the coordination term in the control is a nonlinear function of bearing angle.

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In this paper, a target tracking strategy is proposed where a group of UAVs encircle a target point. The objective is to have the vehicles placed uniformly around the target while they move on a circle of given radius. The control law is simple to compute. The measurements necessary to compute the control are easily available. The law is decentralised and when implemented on real hardware performs as close as it is predicted using unicycle kinematics. In addition to these objectives, we achieve the capability to change the formation while being in the mission in a decentralised manner. The primary contributions of this work are:

- A simple control law based on bearing angle information only.
- Scale the formation by manipulating a parameter of any one of the vehicles.
- Implement the strategy on 6-DOF dynamical model of UAVs.

Concepts from cyclic pursuit, deviated pursuit and consensus are used to achieve the objectives. The paper is organized as follows. The system model with the proposed law is presented in Sec II followed by the discussion about possible equilibrium formations in Sec III. Section IV gives the realistic MAV model and its implementation with HILS. Simulation results are presented in Sec V and concluding remarks are discussed in Sec VI.

II. PROBLEM FORMULATION

Cyclic pursuit is a simple strategy derived from the behavior of social insects. Given a set of n agents, they are numbered from 1 to n and each agent i follows its neighbor agent $i+1$ (mod n). This results into different types of patterns depending on the model of each agent and the way each agent pursues its neighbor. This strategy and its applications has been discussed in great detail in [17] - [24]. In this paper cyclic pursuit strategy has been modified to track a stationary target with a group of n unicycle type agents. The kinematics of each agent i are represented by:

$$\dot{x}_i = V_i \cos(h_i), \quad \dot{y}_i = V_i \sin(h_i), \quad \dot{h}_i = u_i = \omega_i, \quad (1)$$

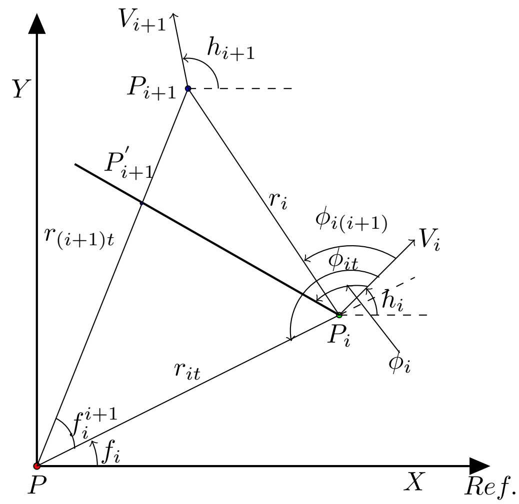


Fig. 1. Positions of the vehicles in a target centric frame.

where $P_i = [x_i, y_i]^T$ represents the position of agent i and h_i represents the heading angle of the agent i. V_i and ω_i represents the linear speed and angular speed of agent i respectively. Equation (1) can represent a point mass model of a UAV flying at a fixed altitude or a point mass model of a wheeled robot on a plane. We use a generic term "agent" to represent the aerial or ground vehicle. We assume that the agent i is moving with constant linear speed, that is V_i is constant and the motion of the agent i is controlled using the angular speed, ω_i.

We start with the assumption that all the agents can see the target during their entire maneuver. But later we show that it can be relaxed and the minimum requirement is that the target is in the vicinity of atleast one agent. The strategy proposed in this paper demands only bearing angle information which can be acquired easily with vision sensors. Consider Fig. 1. Point P in the Fig. 1 represents the target position. Since we are considering a stationary target, we assume a target centric reference frame. Agent i and its neighbor agent $i+1$ are located at P_i and P_{i+1} respectively. The variables in Fig. 1 are:
r_{it} – Distance between i^{th} agent and the target,
r_i – Distance between i^{th} agent and $i+1^{th}$ agent,
f_i – angle made by the vector r_{it} w.r.t reference and,
f_i^{i+1} – angular separation between agent i and agent $i+1$ taken with respect to target,
ϕ_{it} – Bearing angle of agent i with respect to the target,
$\phi_{i(i+1)}$ – Bearing angle of agent i with respect to neighbor agent $i+1$ (mod n).

We modify the classical cyclic pursuit law [19] for target enclosing problem such that agent i, positioned at P_i, follows not only $i+1^{th}$ agent at P_{i+1} but also the target at P. Let ρ_i be a constant which decides the weight agent i gives to the target information over the information of the agent $i+1$. We call this parameter as *pursuit gain*. The parameter ρ_i can take values between 0 and 1. This weighing scheme is mathematically equivalent to following a virtual leader along the line $P_iP'_{i+1}$ with bearing angle ϕ_i. The angle ϕ_i is calculated as

$$\phi_i = (1-\rho_i)\ \phi_{it} + \rho_i\ \phi_{i(i+1)}. \tag{2}$$

We define the control input to the i^{th} agent as

$$u_i = \omega_i = k_i(\phi_i - \delta_i), \tag{3}$$

where, $k_i > 0$ is the controller gain and δ_i is the deviation angle. We update the deviation angle of all the agents as

$$\dot{\delta}_i = \kappa_i(\delta_{i+1} - \delta_i). \tag{4}$$

We assume that the measurement $(\phi_i - \delta_i) \in [0, 2\pi)$. This condition ensures that the agents always rotate in counter clockwise direction.

Let us define the states of the system as $\mathbf{x}_{i(1)} = r_{it}$, $\mathbf{x}_{i(2)} = f_i^{i+1}$, $\mathbf{x}_{i(3)} = h_i - f_i$, $\mathbf{x}_{i(4)} = \delta_i$ for $i = 1, 2, \cdots n$. Then, the kinematics (1) can be re-written in the target centric reference frame as,

$$\dot{\mathbf{x}}_{i(1)} = V_i \cos(\mathbf{x}_{i(3)}), \tag{5a}$$

$$\dot{\mathbf{x}}_{i(2)} = \frac{V_{i+1}\sin(\mathbf{x}_{i+1(3)})}{\mathbf{x}_{i+1(1)}} - \frac{V_i \sin(\mathbf{x}_{i(3)})}{\mathbf{x}_{i(1)}}, \tag{5b}$$

$$\dot{\mathbf{x}}_{i(3)} = k_i(\phi_i - \delta_i) - \frac{V_i \sin(\mathbf{x}_{i(3)})}{\mathbf{x}_{i(1)}}, \tag{5c}$$

$$\dot{\mathbf{x}}_{i(4)} = \kappa_i(\delta_{i+1} - \delta_i). \tag{5d}$$

Equation (5) gives the kinematics of i^{th} agent. In the subsequent sections, all the analysis are done based on this model.

III. Formation at equilibrium

In this section, we study the asymptotic behavior of the agents under the control law (3).

Theorem 1: Consider n agents with kinematics (5) and control law (3). At equilibrium the agents move on concentric circles with rigid polygonal formation.

Proof: At equilibrium $\dot{\mathbf{x}}_{i(j)} = 0$ for $i = 1, \cdots, n$ and $j = 1, 2, 3, 4$. Since (5d) is decoupled from (5a)-(5c), δ_i evolves independent of the other states. From [20], we can state the equilibrium value of δ_i as

$$\delta_{eq} = \sum_{i=1}^{n} \frac{1/\kappa_i}{\sum_{i=1}^{n} 1/\kappa_j}\delta_i(0). \tag{6}$$

From (5a) we get,

$$h_i - f_i = (2m+1)\frac{\pi}{2}, \tag{7}$$

where $m = 0, \pm1, \pm2, \cdots$. From (5c) and (7),

$$k_i(\phi_i - \delta_f) = \frac{V_i \sin(h_i - f_i)}{r_{it}} = \pm\frac{V_i}{r_{it}}. \tag{8}$$

Since $k_i > 0$, $V_i > 0$ and $r_{it} \geq 0$, from (8) we get

$$k_i(\phi_i - \delta_f) = \frac{V_i}{r_{it}}. \tag{9}$$

and therefore, in (7), $m = 0, \pm2, \pm4, \cdots$. Assuming $h_i \in [0, 2\pi)$ and $f_i \in [0, 2\pi)$, we get $(h_i - f_i) \in (-2\pi, 2\pi)$.

Therefore $m = 0$ or $m = -2$. From geometry, $m = 0$ and $m = -2$ implies the same angle. Therefore

$$h_i - f_i = \frac{\pi}{2}. \tag{10}$$

From (3) and (9),

$$\omega_i = \frac{V_i}{r_{it}}. \tag{11}$$

Since V_i and r_{it} are constant, ω_i is constant for all i. Therefore all the agents move along a circular path with the target at the center and radius r_{it}. This proves the first part of the theorem.

From equation (5b) and (10), we can write,

$$\frac{V_i}{r_{it}} = \frac{V_{i+1}}{r_{(i+1)t}}. \tag{12}$$

Using (11) and (12), we conclude that

$$\omega_i = \omega_{i+1}, \tag{13}$$

for all i. Therefore, all the agents move around the target in concentric circles with equal angular speed. So at equilibrium the agents form a rigid polygon that rotates about the target. ■

In this paper we present analysis for homogeneous agents only. The agents are assumed to be homogeneous in the sense that all of them move with equal linear speed V and equal controller gain k. As $V_i = V_{i+1}$, from (12), $r_{it} = r_{(i+1)t} = R$ for all i. Therefore at equilibrium all the agents move along a circle of radius R with the target at the center. Consider Fig. 2 which shows the position of two of the agents at equilibrium. Let P, P_i and P_{i+1} be the positions of target, agent i and agent $i+1$ respectively. From Fig. 1, $\phi_{it} = \pi - (h_i - f_i)$. Substituting the value of $h_i - f_i$ from (10),

$$\phi_{it} = \frac{\pi}{2}, \tag{14}$$

for all i. Consider $\triangle P_i P P_{i+1}$. As $P_i P = P_{i+1} P = R$, $\angle P P_i P_{i+1} = \angle P P_{i+1} P_i = b_i$. Therefore $f_i^{i+1} = \pi - 2b_i$. Referring Fig. 2 and using (14), we can write $b_i = \frac{\pi}{2} - \phi_{i(i+1)}$. So

$$\phi_{i(i+1)} = \frac{f_i^{i+1}}{2}. \tag{15}$$

From (3), (11) and (13) we can write,

$$\phi_i = \phi_{i+1} = \frac{V}{kR} + \delta_{eq}, \tag{16}$$

for all i. So using (2), (14), (15) and (16) we can write

$$(1-\rho_i)\frac{\pi}{2} + \rho_i \frac{f_i^{i+1}}{2} = (1-\rho_{i+1})\frac{\pi}{2} + \rho_{i+1}\frac{f_{i+1}^{i+2}}{2},$$

for all i. Rearranging this equation we can write

$$f_i^{i+1} = \left(\frac{\rho_i - \rho_1}{\rho_i}\right)\pi + \frac{\rho_1}{\rho_i} f_1^2. \tag{17}$$

As the agents are distributed along a circle,

$$\sum_{\substack{i=1 \\ (\mathrm{mod}\ n)}}^{n} (f_i^{i+1}) = 2\pi d,$$

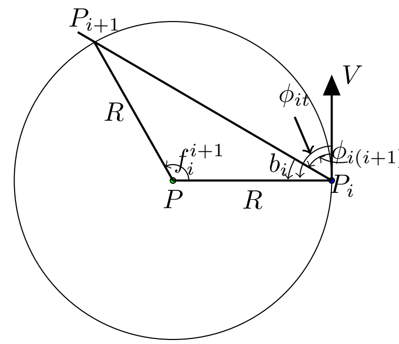


Fig. 2. Formation of agent i and agent $i+1$ at equilibrium.

where $d = 0, \pm 1, \pm 2, \cdots$. Expanding this equation and substituting the value of f_i^{i+1} (from equation (17)) in terms of f_1^2, we get,

$$f_1^2 = \frac{(2\pi d - n\pi)\rho_{eq} + \pi\rho_1}{\rho_1}, \tag{18}$$

where $\frac{1}{\rho_{eq}} = \frac{1}{\rho_1} + \frac{1}{\rho_2} + \cdots + \frac{1}{\rho_n}$. Using (17) and (18) we can calculate

$$f_i^{i+1} = \frac{(2\pi d - n\pi)\rho_{eq} + \pi\rho_i}{\rho_i}, \tag{19}$$

for all i. The value of ϕ_i at equilibrium can be calculated by using (2), (14), (15) and (18) as,

$$\phi_{eq} = (1 - n\rho_{eq})\frac{\pi}{2} + \rho_{eq}\pi d. \tag{20}$$

When we consider all the agents with equal ρ that is $\rho_i = \rho_{i+1}$ for all i, then the value of $\rho_{eq} = \frac{\rho}{n}$. Also the inter-agent angular separation will be $f_i^{i+1} = f_{i+1}^{i+2} = 2\pi\frac{d}{n}$. So the equilibrium state of the system can be described as

$$\mathbf{x}_{i(1)} = R = \frac{V}{k(\phi_{eq} - \delta_{eq})}, \tag{21a}$$

$$\mathbf{x}_{i(2)} = 2\pi\frac{d}{n}, \tag{21b}$$

$$\mathbf{x}_{i(3)} = \frac{\pi}{2}, \tag{21c}$$

$$\mathbf{x}_{i(4)} = \sum_{i=1}^{n} \frac{1/\kappa_i}{\sum_{i=1}^{n} 1/\kappa_j}\delta_i(0), \tag{21d}$$

where

$$\phi_{eq} = (1-\rho)\frac{\pi}{2} + \rho\pi\frac{d}{n}. \tag{22}$$

Thus, at equilibrium, the agents arrange themselves in a regular formation around the target. This regular formation of n agents can be described by a regular polygon $\{n/d\}$, where $d \in \{1, 2, ..., n-1\}$.

Corollary 1: The equilibrium of n agents with kinematics (5) and control law (3) will exist if the initial deviation angle $\delta_i(0)$ satisfy $\sum_{i=1}^{n} \frac{1/\kappa_i}{\sum_{i=1}^{n} 1/\kappa_j}\delta_i(0) < (1-\rho)\frac{\pi}{2} + \rho\pi\frac{d}{n}$.

Proof: This follows from (21a) since $V > 0$, $k > 0$ and $R > 0$. ■

In many applications it is required to move the vehicles near to or far away from the target by compressing or expanding the existing formation to the desired formations.

Next we will achieve this objective by changing only one the values of δ_i.

Theorem 2: For a system of n agents with kinematics (5) and control law (3) at equilibrium with $\delta_{eq} = \delta_f$, the radius of the circle of enclosure can be changed to R_{cn} by changing δ_k for any $k \in \{1, \ldots, n\}$ as

$$\delta_k = \left((1-\rho)\frac{\pi}{2} + \rho\pi\frac{d}{n} - \frac{V}{kR_{cn}}\right)\sum_{i=1}^{n}\frac{\kappa_k}{\kappa_i} - \delta_f \sum_{i=1, i\neq k}^{n}\frac{\kappa_k}{\kappa_i}. \tag{23}$$

Proof: From (21a) and (22), the new value of δ_{eq} required to obtain R_{cn} is given as $\delta_{eq} = (1-\rho)\frac{\pi}{2} + \rho\pi\frac{d}{n} - \frac{V}{kR_{cn}}$. Then, using (6), δ_k can be calculated as given in (23) ■

We can decide a switching strategy for deciding the value of ρ depending on the availability of information about the target. It is assumed that information topology is fixed among the agents and the target is in the sensing range of atleast one agent. Let ρ_g be the value of pursuit gain if an agent has the information about the target. Then with the limited information, the strategy can be implemented as follows:

- If an agent is able to sense the target set ρ to a group pursuit gain ρ_g.
- If an agent is not able to sense the target set $\rho = 1$.
- If the vision sensor is able to give range measurement then we take ρ inversely proportional to distance between the target and agent up to certain distance and once they are close enough it can be set to ρ_g.

This algorithm is useful when the information is limited in the sense that only few of the agents able to see the target.

IV. Implementation with 6-DOF Model in HILS

In this section, we discuss implementation of the proposed strategy for fixed-wing MAVs. The flight model is taken from [27], in which the wind tunnel data was obtained from National Aerospace Laboratories, Bangalore. The aerodynamic equations used are as follows:

$$\begin{aligned}
\dot{x}_e &= [u\cos\theta + (v\sin\phi + w\cos\phi)\sin\theta]\cos\psi \\
&\quad - (v\cos\phi - w\sin\phi)\sin\psi, \\
\dot{y}_e &= [u\cos\theta + (v\sin\phi + w\cos\phi)\sin\theta]\sin\psi \\
&\quad + (v\cos\phi - w\sin\phi)\cos\psi, \\
\dot{z}_e &= -u\sin\theta + (v\sin\phi + w\cos\phi)\cos\theta, \\
\dot{\theta} &= q\cos\phi - r\sin\phi, \quad \dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta, \\
\dot{\psi} &= \frac{(q\sin\phi + r\cos\phi)}{\cos\theta}, \quad \dot{u} = rv - qw + \frac{1}{\mathrm{m}}f_x, \\
\dot{v} &= pw - ru + \frac{1}{\mathrm{m}}f_y, , \quad \dot{w} = qu - pv + \frac{1}{\mathrm{m}}f_z, \\
\dot{p} &= \Gamma_1 pq - \Gamma_2 qr + \Gamma_3 l + \Gamma_4 n, \\
\dot{q} &= \Gamma_5 pr - \Gamma_4\left(p^2 - r^2\right) + \Gamma_5 m, \\
\dot{r} &= \Gamma_6 pq - \Gamma_1 qr + \Gamma_4 l + \Gamma_7 n,
\end{aligned}$$

where $[x_e, y_e, z_e]$ represents position of MAV, $[u, v, w]$ represents velocity components of MAV in body frame. Here $[\phi, \theta, \psi]$ and $[p, q, r]$ are Euler angles and body axis angular rates respectively. Also $[l, m, n]$ are roll, pitch and yaw moments. The autopilot design and the HILS are discussed in [27].

TABLE I

SIMULATION RESULTS WITH $\rho = [0.9 \quad 0.8 \quad 0.7]$

	Point mass	6-DOF	HILS
d	1	1	1
Radius	95.22	101.79	102.60
f_1^2	127.22	127.28	134.85
f_2^3	120.63	119.98	116.74
f_3^1	112.15	112.74	108.34

TABLE II

RADIUS OF THE CIRCLE WITH SAME VALUES OF ρ FOR ALL AGENTS WITH THREE DIFFERENT IMPLEMENTATIONS

		Point mass	6-DOF		HILS	
rho	d	Radius	Radius	Error(%)	Radius	Error(%)
0.1	1	72.53	76.11	4.93	70.12	3.32
0.2	1	75.12	77.06	2.58	72.50	3.49
0.3	1	77.91	79.29	1.78	74.96	3.77
0.4	1	80.90	82.59	2.08	78.41	3.07
0.5	1	84.14	86.14	2.38	83.34	0.95
0.6	1	87.64	89.98	2.66	87.27	0.42
0.7	1	91.45	94.14	2.93	92.21	0.83
0.8	1	95.61	98.76	3.29	97.22	1.68
0.9	1	100.16	104.53	4.35	102.61	2.44

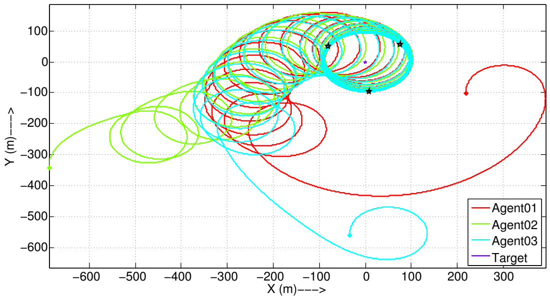
V. Simulation Results

A group of three agents has been considered, moving with a linear speed of 14.7 m/sec and having controller gain $k = 0.2$. The target is located at the origin $(0, 0)$ and the vehicles start from random initial positions. Simulations are run with different values of pursuit gain, $\rho = [0.9\ 0.8\ 0.7]$ and $\delta_i(0) = 0$ for all i. Fig. 3 shows the trajectories of the vehicles with unicycle model, 6 - DOF model and HILS respectively starting from same initial conditions. The vehicles are able to capture the target with nonuniform distribution. From Table I it can be observed that the final radius of the circle and the angular separation between the agents matches closely in all three implementations. Then, we have considered the same pursuit gain for all the agents i.e. $\rho = 0.5$. Table II shows different parameters at steady state for different values of ρ. It can be seen that the error in the radius is within 5% of the analytical value.

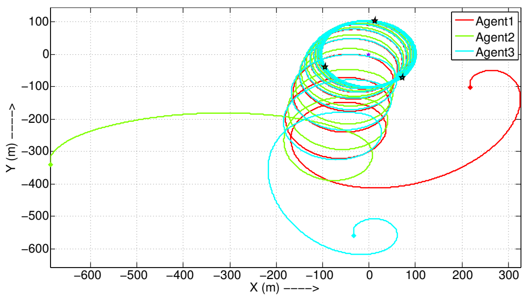
Next, we have considered $\delta(0) = [5°,\ 5°,\ 0°,\ 0°,\ 0°]$ and $\kappa_i = 1$ for all i. At $t = 300sec$, δ_1 is change to $\delta_1 = 353.46°$ such that formation expand to new radius $R_{cn} = 130$. Fig. 4 shows the trajectory, evolution of delta, inter-agent distances and target to agent distances before and after δ_1 is changed and it is confirmed from simulation that the desired change in formation is achieved. The same experiment is repeated with the intention to compress the radius to $R_{cn} = 60$ and the results are presented in Fig. 5.

VI. Conclusions

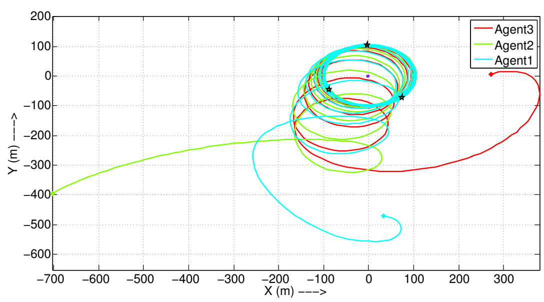
In this paper, a deviated cyclic pursuit control strategy has been proposed for target tracking applications. As the name suggests, the connection among the agents is cyclic. The controller requires only bearing angle information of the



(a) Unicycle model



(b) 6-DOF MAV model



(c) HILS

Fig. 3. Trajectories of three vehicles with different ρ.

target and the neighbouring agent. The angle of deviation is initially different for different agents, but they eventually reach a consensus. The deviation angle influence the radius of the circle formed around the target. This motived us to abruptly change the deviation angle of one of the agents to initiate the change in radius. The formation at equilibrium is studied using unicycle kinematic and simulations are presentation for 6DOF model of UAVs and in hardware-in-loop simulators. The simulation results match closely with the analysis in case of target tracking without δ consensus. The immediate future work is to implement the system with δ consensus in HILS. The implementation of the algorithm in actual hardware will be the ultimate goal of this work.

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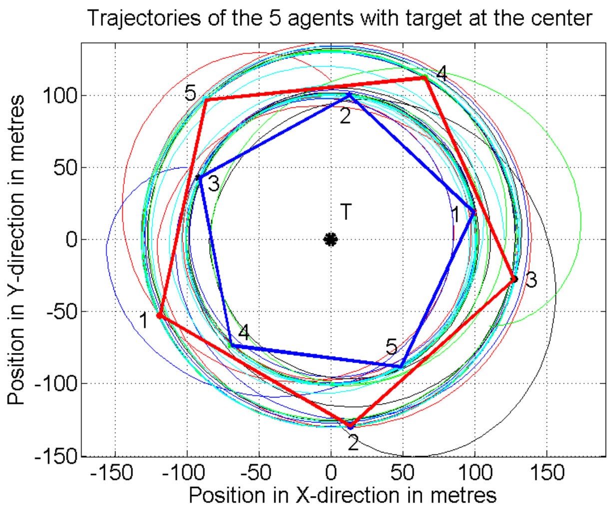
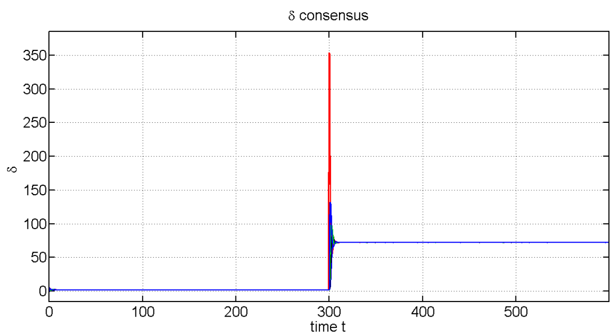
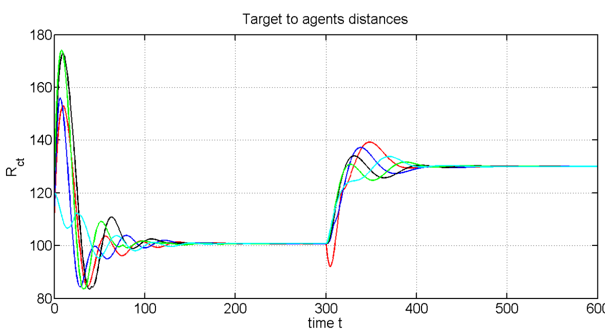


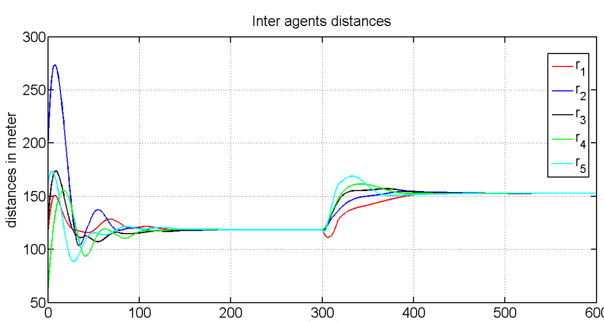
(a) Trajectories of the vehicles



(b) The evolution of δ_i, $i = 1, \dots, n$ with time

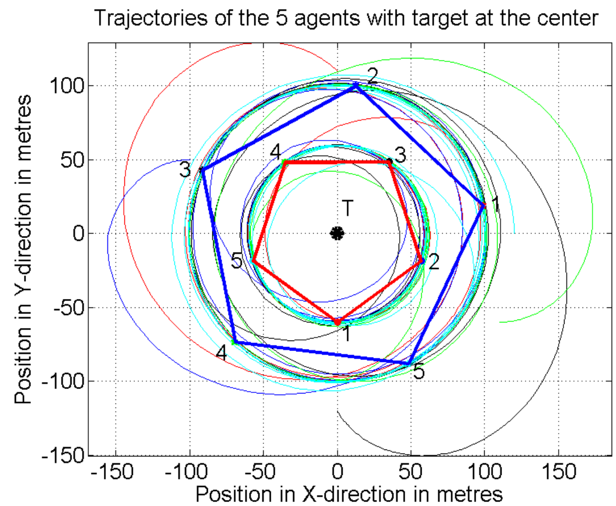


(c) Radius of encirclement

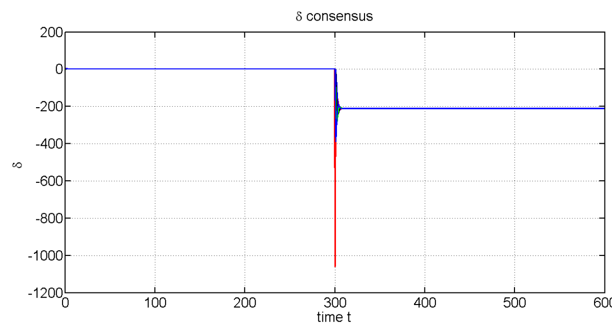


(d) Interagent distances

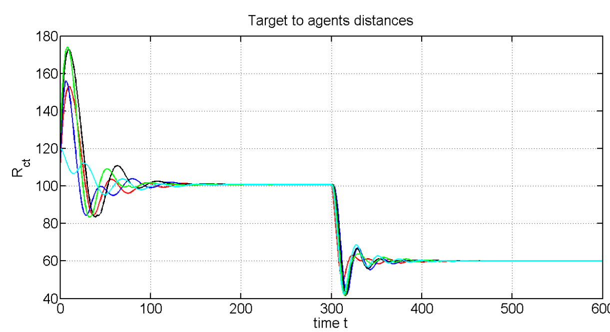
Fig. 4. Changing δ_1 to move away from the target.



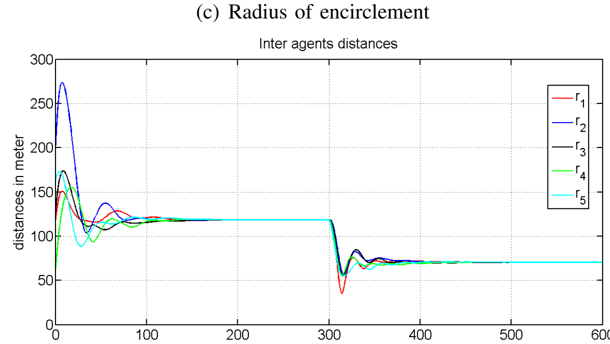
(a) Trajectories of the vehicles



(b) The evolution of δ_i, $i = 1, \dots, n$ with time



(c) Radius of encirclement



(d) Interagent distances

Fig. 5. Changing δ_1 to close on to the target.