A Study of Balanced Circular Formation under Deviated Cyclic Pursuit Strategy

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Abstract: This paper deals with the study of balanced circular formation for n unicycles agents. It has been presented in the literature that there exists more than one balanced circular formation when the agents are following cyclic pursuit strategy. We modify cyclic pursuit strategy by introducing a parameter called deviation angle with the hope to get unique equilibrium formation. The modified strategy is called deviated cyclic pursuit. Monte Carlo simulations has been carried out to find the range of deviation angle where a unique equilibrium formation exists.

Keywords: Co-operative control, Co-ordination, Co-operation, Navigation systems, Guidance systems, Autonomous control.

1. INTRODUCTION

Formation control of a group of agents are nowadays topics of increasing popularity. It is sometimes desired to achieve a formation with certain geometric shape. In this scenario, we are particularly interested about the geometric pattern formation control problems. Also we are interested in the control strategy which is simple and easy to implement in real world applications. Cyclic pursuit is a simple control strategy which involve a local interaction between the agent and its neighboring agent. In cyclic pursuit, the control strategy in which n agents are placed in such a way that the agent i pursues agent $i+1$ modulo n. The formation control strategies for group of agents under cyclic pursuit have been investigated in the following literature.

Lin et al. (2004) has proposed the linear cyclic pursuit algorithm for a system of identical agents under which all the agent will converge to a point. In Marshall et al. (2004), authors extended the classical cyclic pursuit to a system of wheeled vehicles each having unicycle model. They showed formations are generalized regular polygons at equilibrium and analyzed the local stability of those equilibrium polygons. Sinha and Ghose (2007) has generalized the problem of formation of group of autonomous agents by considering nonlinear cyclic pursuit. They derived a necessary condition at equilibrium formation by considering different speed and controller gain.

Galloway et al. (2009) has presented differential geometric control approach to study the geometry of cyclic pursuit. They also stated necessary and sufficient conditions for the existence of rectilinear and circular formation.

Morbidi et al. (2010) has studies the connectivity maintenance problem in linear and nonlinear cyclic pursuit when different control gains are assigned to each agent.

Ramirez-Riberos et al. (2010) has study cyclic pursuit control laws for both single and double integrator models in three dimensional plane. They developed control law that only require relative measurements of position and velocity with respect to the two leading neighbors in the ring topology of cyclic pursuit. In W. Ding and Lin (2010), authors presented a pursuit based approached to achieve formation of group of agents having both single-integrator kinematics and double-integrator dynamics on directed acyclic graphs.

Ma and Hovakimyan (2013), have discussed coordinated control law for a group of agents that can track moving target with known velocity. Further they have discussed vision based cooperative target tracking when the target's motion information are unknown. Daingade and Sinha (2014), proposed control strategy to track stationary target with multiple agent. Juang (2014), have proposed generalized cyclic pursuit control law to achieve formation pattern that are characterized in terms of epicyclic motions. The author incorporated a rotation operation and stabilization term to the the classical cyclic pursuit control algorithm. Tao et al. (2014) have proposed a methodology for a group of agents to achieve formation reconfiguration and phase angle adjustment. The proposed approach consists of planner movement control and orthogonal displacement control. Planner movement is based on cyclic pursuit strategy and for phase angle adjustment the control is based on beacons guidance strategy.

Marshall et al. (2004), Sinha and Ghose (2007) and Daingade and Sinha (2014) have showed that the balanced circular formation can be achieved with different formulation of cyclic pursuit strategy. They showed multiple equilibrium points corresponds to different formation that can be achieved. The final formations are dependent on the initial conditions. Hence it is not possible to predict any unique formation for a given initial condition. This motivates us to consider this problem, so that we can achieve a unique formation irrespective of any initial condition. We have introduced a parameter called deviation angle in the control law in an earlier paper Mallik and Sinha (2015), where we are able to vary the inter-agents distance between the agents with the help of δ. In this paper, we want to find a range of δ where we can achieve unique formation.

The paper organized as follows. Section 2 gives the problem formulation for n agents with each having unicycle model. In Section 3, we have studied and established a range of deviation angle where a unique equilibrium formation exists. Various

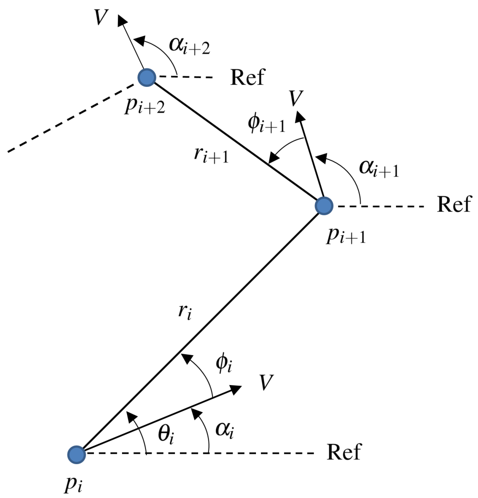


Fig. 1. Formation geometry of n agents.

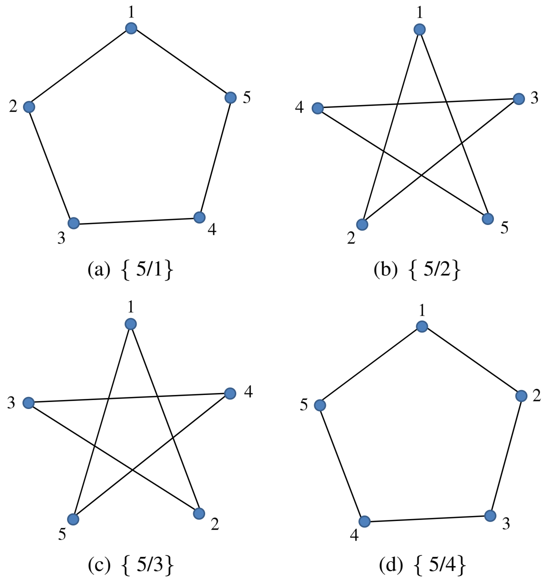


Fig. 2. Possible formations type with n=5 agents.

equilibrium formations are provided. Finally, conclusions and future works are provided in Section 4.

2. PROBLEM FORMULATION

We consider n unicycle agents with kinematics

$$\dot{x}_i = V\cos(\alpha_i), \dot{y}_i = V\sin(\alpha_i), \dot{\alpha}_i = u_i, \tag{1}$$

where (x_i, y_i) is the position of the agent i, α_i is the orientation with respect to some inertial reference frame. We assume that all agents are moving with the same and constant linear speed V. We control the angular velocity of the agents. The agents are in cyclic pursuit, that is, the agent i follows agent $i+1$, modulo n. This is depicted in Figure 1, where p_i is the position of agent i. Let $\phi_i = \theta_i - \alpha_i$. We define the controller for agent i as

$$u_i = k(\phi_i - \delta), \tag{2}$$

where δ is a deviation angle and it is assumed to be a constant. All agents have the same controller gain k.

The system of n agents with control law (2) has been studied in (Mallik and Sinha (2015)). We present the result here for easy referencing.

Theorem 1. (Mallik and Sinha (2015)) At equilibrium, a system consisting of n agents with kinematics (1) and control 2 will move on the same circular trajectory.

Therefore at equilibrium all the agents will move on a circular trajectory with same radius R_c. R_c and inter agents distance $\bar{r}_i$, for all i are derived in (Mallik and Sinha (2015)) and reproduced here

$$R_c = \frac{V}{k[\pi\frac{d}{n} - \delta]}, \tag{3}$$

$$\bar{r}_i = 2\frac{V}{k[\pi\frac{d}{n} - \delta]}\sin(\pi\frac{d}{n}), \tag{4}$$

where $d = 0, \ldots, n-1$.

δ is the design parameter. It has been shown in (Mallik and Sinha (2015)) that δ must satisfy the following condition

$$\delta < \pi\frac{d}{n}. \tag{5}$$

The formation type is characterized by d and can have either regular or star type depending on the values of d as shown in figure 2. Therefore following Marshall et al. (2004) define a generalized regular polygon as

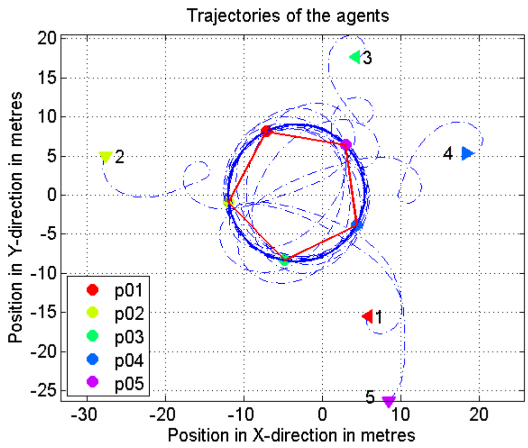
Let n and $d < n$ be positive integers so that $p := n/d > 1$ is a rational number. Let R be the positive rotation in the plane, about the origin, through angle $2\pi/p$ and $z_1 \neq 0$ be a point in the plane. Then, the points $z_{i+1} = Rz_i$, $i = 1, 2, \ldots n-1$ and edges $e_i = z_{i+1} - z_i$, $i = 1, 2, \ldots n$, be define a *generalized regular polygon*, which is denoted by { p}.

Definition 2. A formation pattern is define as formation of type {n/d} for $d = 1, \ldots, n$.

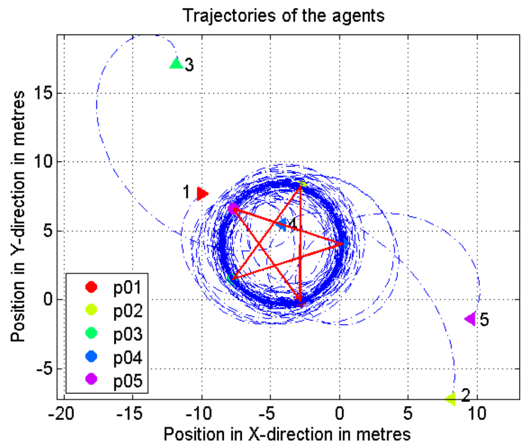
Let us take an example with $n = 5$. When d $= 1$, {5/1} is regular polygon as shown in figure 2(a). If $d > 1$ and is coprime to n, then {n/d} is a star polygon. With $d = 2$, $d = 3$ and $d = 4$ formation {5/2} , {5/3} and {5/4} are star polygon which are shown in figure 2(b), 2(c) and 2(d) respectively. If n and d have common factor $s > 1$, then {n/d} has $t = n/s$ distinct vertices and t edges traversed s times.

There are many applications where it is required to achieve circular formation with desired radius. From (3), we know that the radius is dependent on d. Hence it is necessary to have a specific formation type in order to achieve circular formation with desired radius. Current literature (Marshall et al. (2004), Sinha and Ghose (2007)) conveys that the formation pattern is dependent on the initial conditions. Therefore it is not possible to achieve a desired circular radius from any initial position. Let us illustrate this with figure 3. Figure 3(a) shows the simulation result for a system of $n = 5$ agents and they all starts from any random initial position and heading angle under the control law (2) with $\delta = 0$. We can observe that the agents achieve {5/1} formation. Similarly figure 3(b) shows the simulation results where agents start with another initial position and heading angle under the same control law (2) with $\delta = 0$. But this time agents achieve {5/2} formation pattern. This results were shown in Marshall et al. (2004). Therefore we can say that it is not possible to get a desired formation pattern from any initial position.

The main objective of this paper as follow. Firstly, if we vary δ, can we have a control over d?. Secondly, if so, then we look for range of δ.



(a) { 5/1} formation



(b) { 5/2} formation

Fig. 3. Trajectories of five agents subject to control law (2) with $\delta = 0$ (Marshall et al., 2004).

3. FORMATION AT EQUILIBRIUM

From (3), it can be seen that for a given k, V and δ there are $(n-1)$ different equilibrium possible and so $(n-1)$ possible values of R_c. The radius achieved is dependent on the formation type $\{n/d\}$. As we have mentioned earlier, that the formation type depends on the initial positions and heading angle. We do not have any control over d and that makes it difficult to predict any formation type. Therefore it is utmost important to have control over d in order to achieve desired circular radius at equilibrium. We are interested to see the effect of δ in the formation types at equilibrium.

We are trying to understand and analyze the behavior of formation types at equilibrium based on Monte Carlo simulations. We have performed the simulation in the following way. We consider random initial position in a square area of $(-20,20) \times (-20,20)$ units. We have consider the range of δ as $-0.9\frac{\pi}{n} \leq \delta \leq 0.9\frac{\pi}{n}$. Let us define $\Delta \in [-0.9, 0.9]$ as

$$\delta = \Delta \frac{\pi}{n}. \tag{6}$$

We considered $n = 4, \ldots, 10$ agents. For each n, we consider nine different values of $\Delta = \{-0.9, \ -0.7, \ldots, 0.7, \ 0.9\}$. For each Δ, we consider 100 different initial conditions and observed the final formation. The frequency of occurrence of different d are noted in the bar graph figure 4. We observed in figure 4(a) that for $n = 4$ and $\Delta \in [0.3, 0.9]$, $d = 1$ occurred for all the 100 initial conditions. Similarly in figure 4(b)-4(g), $d = 1$ always, occurred when $\Delta \in [0.3, 0.9]$. However, for other values of Δ, more than one value of d is occurring where we can conclude that we will get $d = 1$, if $\Delta \in [0.3, 0.9]$ for any n.

Next, we give the details of some of the simulations carried out. In all the cases we have consider linear speed and controller gain as $V = 14.7$ and $k = 0.1335$ respectively.

Case-1: For $n = 5$ agents, all the agents starts from random initial position and heading angle. Figure 5(a) shows that the agents achieved formation type $\{5/2\}$ when $\delta = -0.9\frac{\pi}{5}$, where as figure 5(b) shows the formation type $\{5/1\}$ for the same initial conditions when $\delta = 0.9\frac{\pi}{5}$.

Case-2: Here we have considered $n = 7$ agents. Here, all agents starts from random initial position and heading angle. Under the control law (2) with $\delta = 0.1\frac{\pi}{7}$, all the agents achieved formation type $\{7/2\}$ as shown in figure 5(c). Now if all the agents starts from the same initial conditions we get formation type $\{7/1\}$ under the control law (2) with $\delta = 0.3\frac{\pi}{7}$ as shown in figure 5(d) shows .

Case-3: Here we have considered $n = 8$ agents. Figure 5(e) shows formation type $\{8/3\}$ under the control law (2) with $\delta = -0.5\frac{\pi}{8}$, where as figure 5(f) shows the formation type $\{8/1\}$ under the control law (2) with $\delta = 0.4\frac{\pi}{8}$.

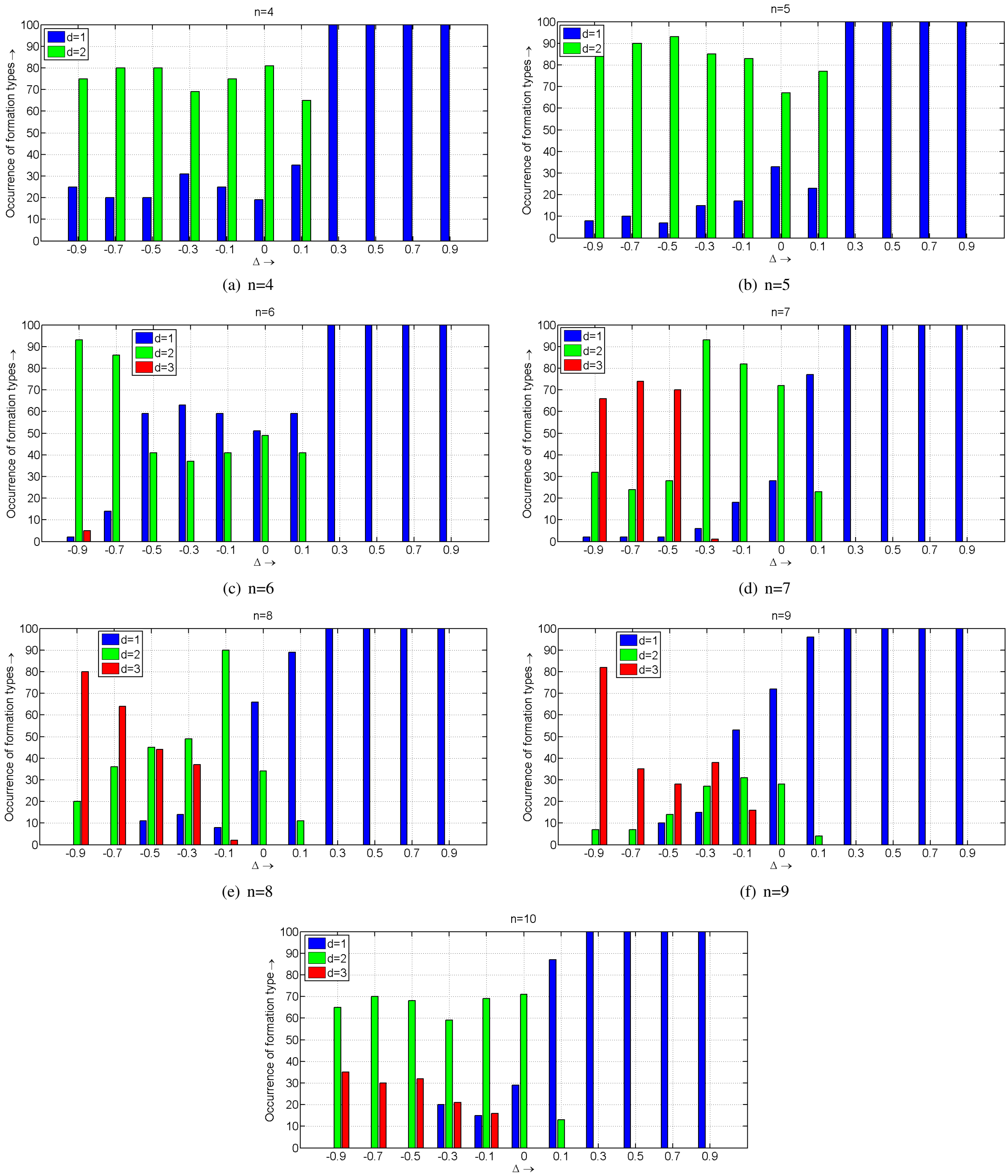
Case-4: Here we have considered $n = 10$ agents. Figure 5(g) shows formation type $\{10/3\}$ under the control law (2) with $\delta = -0.7\frac{\pi}{10}$, where as figure 5(h) shows the formation type $\{10/1\}$ under the control law (2) with $\delta = 0.7\frac{\pi}{10}$.

Therefore all the simulations results confirm that the system of agents can only achieve formation type {n/1} if $\delta \in [0.3\frac{\pi}{n}, 0.9\frac{\pi}{n}]$ for all n.

Case-5: This case illustrate the importance of a specific formation type. Consider a system of $n = 5$ agents. Suppose we want to achieve a circular formation with desired circular radius $R_c = 100$ with formation type $\{5/1\}$. We set the linear speed and control gain of the each agent be $V = 20m/s$ and $k = 0.2$ respectively. Therefore from (3), we must have $\delta = -0.37$. Figure 6(a) shows that the agents are unable to achieve the desired circular radius. Hence in order to achieve this radius we have to change the value of δ such that it belongs to the range $0.3\frac{\pi}{n} \leq \delta \leq 0.9\frac{\pi}{n}$. This can be done by changing the control gain (k) of the vehicle. With $k = 0.64$ and rest of the parameters are same as before. The new value of δ is $\delta = 0.5\frac{\pi}{5} = 0.314 rad$, which is inside the desired range. Simulation results in figure 6(b) shows that the agents are able to achieve the desired circular formation with the desired radius.

4. CONCLUSION

For a system of n agents, there exists $(n-1)$ equilibrium points. Formations at the equilibrium are characterized by the formation types. The formation types are dependent on initial conditions. Hence it is difficult to analyze mathematically, how the formation type is dependent on the initial conditions. We carried out Monte Carlo Simulations for $\delta \in [-0.9\frac{\pi}{n}, 0.9\frac{\pi}{n}]$. It is observed that if $\delta \in [0.3\frac{\pi}{n}, \frac{\pi}{n})$, we will always get {n/1} formation type for all n. This has been validate from $n = 4, \ldots, 10$ agents. This result ensure that the agents achieve desired polygonal formation with the required radius. Future work



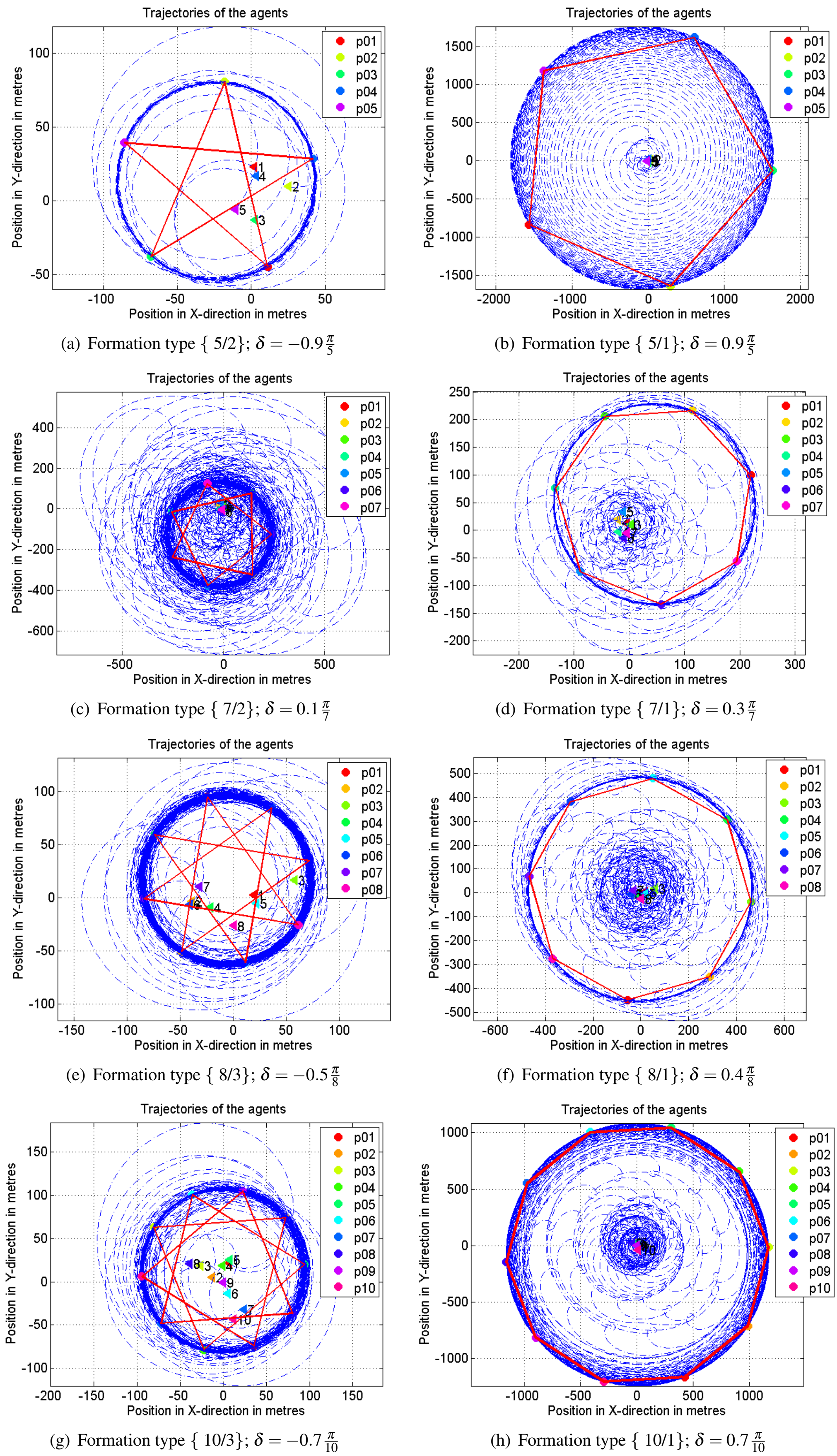
(a) n=4 (b) n=5

(c) n=6 (d) n=7

(e) n=8 (f) n=9

(g) n=10

Fig. 4. Frequency of occurrence of different formation type.



(a) Formation type { 5/2}; $\delta = -0.9\frac{\pi}{5}$

(b) Formation type { 5/1}; $\delta = 0.9\frac{\pi}{5}$

(c) Formation type { 7/2}; $\delta = 0.1\frac{\pi}{7}$

(d) Formation type { 7/1}; $\delta = 0.3\frac{\pi}{7}$

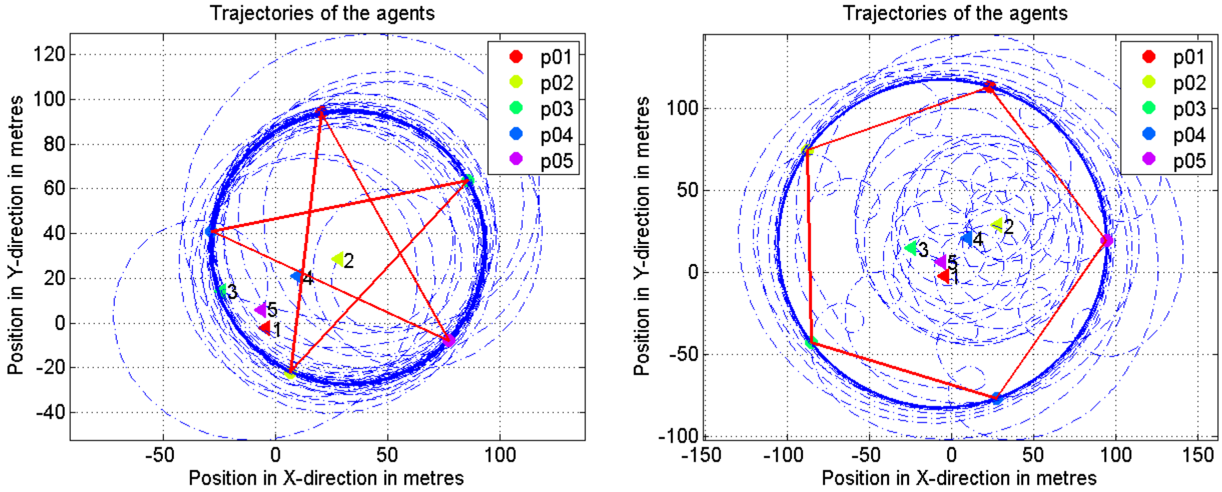
(e) Formation type { 8/3}; $\delta = -0.5\frac{\pi}{8}$

(f) Formation type { 8/1}; $\delta = 0.4\frac{\pi}{8}$

(g) Formation type { 10/3}; $\delta = -0.7\frac{\pi}{10}$

(h) Formation type { 10/1}; $\delta = 0.7\frac{\pi}{10}$

Fig. 5. Equilibrium formation for different values of δ.



(a) $\delta = -0.37rad; k = 0.2$

(b) $\delta = 0.314rad; k = 0.64$

Fig. 6. Trajectories of $n = 5$ agents required to achieve $R_c = 100$.

will involve by similar exercise when δ_i is different for different agents.

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