

# Capturing a target with range only measurement

Gaurav Chaudhary<sup>1</sup> and Arpita Sinha<sup>2</sup>

**Abstract**—This paper addresses the problem of guiding a mobile robot towards a target using only range sensors. The bearing information is not available. The target can be stationary or moving. It can be the source of some gas leakage or nuclear radiation or it can be some landmark or beacon or any maneuvering vehicle. The mobile robot can be ground or aerial vehicle. In literature, many different strategies are proposed which use the range only measurement but they involve estimation of different parameters or have switching control strategy which makes them difficult to implement. We have proposed a strategy that can bring the robot arbitrarily close to the moving target. It is shown analytically that this strategies works for any initial condition of the robot with respect to the target. The results are validated through simulation.

## I. INTRODUCTION

When an autonomous vehicle pursues a stationary or moving target, it is known as a target tracking problem. Strategies for the mobile robots to capture a target usually require both the range(distance) information and bearing(direction) information of target with respect to the robot. Recently the research on capturing a target using range only measurement has attracted special attention in field of robotics (see e.g.[1], [2], [3] and references therein). The objective here is to find a strategy such that the target can be captured with range only measurement. We can use this strategy in applications such as finding the point of leakage of some gas by only knowing the intensity of gas at the robot position or in pursuit evasion problem using only distance information between the pursuer and the evader.

Several different estimation based techniques are discussed in ([8], [9], [10], [11] and [12]) for tracking of target using range only measurement. In [6], local observability requirements are developed for target tracking and are verified by evaluating the performance of a state estimator. Problem of target motion analysis from range and range-rate measurements is investigated in [5]. Range only Extended Kalman Filter (EKF) is utilized to track the trajectory of the moving target in [7].

This problem is solved in discrete time in [4] where the target position is estimated at each instance of time and the robot moves towards the estimated target position in discrete steps. Some more different strategies are proposed which uses range only information for target tracking without estimating states of the system. In [1], a sliding mode control strategy is proposed using which the robot follows the target

with constant speed while maintaining the predefined range margin from the target. In [2], guidance algorithms for following both steady and moving targets are proposed and the guidance methods have the property that the robot follows a trajectory that is close to a certain curve called equiangular spiral. A switched logic-based control strategy to solve the pursuit problem for target tracking is shown in [3].

For a team of mobile robots tracking a moving target using distance-only measurements, Zhou et al. ([13], [14]) have proposed algorithms for determining the set of feasible locations that each robot should move to in order to collect the most informative measurements, that is, the distance measurements that minimize the uncertainty about the position of the target. A strategy for searching the source of gas using mobile robot is discussed in [16]. When the presence of gas is detected, the robot turns in the direction of the airflow that carries the gas and looks for any suspicious object. In [15], the robot is driven by the concentration gradient generated by a gas leak.

We want to design a control strategy for an autonomous vehicle pursuing a target which can be stationary or moving with a bound on maximum lateral acceleration. This problem is formulated as below:

- The vehicle is modeled as a unicycle with constant speed and bound on the maximum lateral acceleration.
- The target is moving randomly and has a bound on the speed and maximum lateral acceleration.
- The robot can measure only the distance and rate of change of distance between itself and the target.
- The target is assumed to be captured if the robot can reach within a certain distance of the target called the capture distance.
- The control strategy does not involve any estimation of states.

This paper is organized as follows. In Section 2, we define the problem and analyze it in Section 3. Simulation results are presented in Section 4 and Section 5 concludes the paper.

## II. PROBLEM STATEMENT

Consider a mobile robot that can measure the distance and the rate of change of distance from a given point. The problem is to guide the robot to that point. The point may be stationary or moving. We consider a unicycle model for the robot, the kinematics of which is given by

$$\dot{x}_r = v_1 \cos \alpha_1 \quad (1)$$

$$\dot{y}_r = v_1 \sin \alpha_1 \quad (2)$$

$$\dot{\alpha}_1 = u \quad (3)$$

<sup>1</sup>G. Chaudhary is with KPIT Cummins Infosystems Limited, Pune, India gaurav30.chaudhary@gmail.com

<sup>2</sup>A. Sinha is with Faculty of Systems and Control Engineering, Indian Institute of Technology, Bombay, India. arpita.sinha@iitb.ac.in

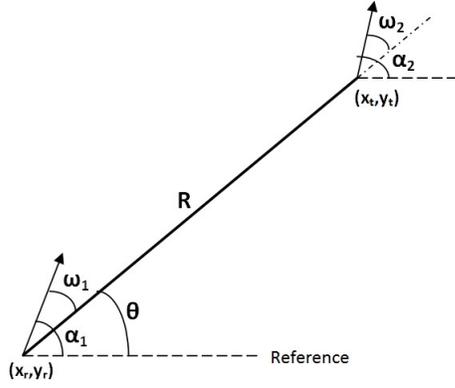


Fig. 1. Engagement geometry between the robot and the target

where  $(x_r, y_r)$  is the instantaneous position of the robot,  $v_1 > 0$  is the linear velocity and  $\alpha_1$  is the heading direction of the robot and  $u$  is the control input to the robot. Thus, the angular velocity of the robot is controlled while the linear velocity is constant. We assume that the lateral acceleration of the robot is bounded as  $\dot{\alpha}_1 \in [\dot{\alpha}_{1 \min}, \dot{\alpha}_{1 \max}]$

We are considering a target that can move randomly in any direction. It has a constant speed  $v_2$  and heading direction  $\alpha_2$ . There is an upper bound on the maximum speed the target can have. Also, we assume that there is a limit on the maximum angular speed the target can have.

Let the target be at  $(x_t(t), y_t(t))$  with respect to same reference frame and the robot be at  $(x_r(t), y_r(t))$  at time  $t$ . This is shown in Figure 1. Let at any instant of time, the distance between the robot and the target, that is, the line-of-sight (LOS) distance be  $R$  and the LOS angle be  $\theta$ . The robot can measure  $R$  and  $\dot{R}$ , but not  $\theta$ . Thus,  $u$  is some function of  $R$  and  $\dot{R}$ . We determine the conditions that this function needs to satisfy so that it can steer the robot to the target. The conditions are derived in the next section. When the distance between the robot and the target is less than  $R_c$ , we assume that the target is captured. We call  $R_c$  the *capture radius* and  $R_c > 0$ .

### III. ANALYSIS

Consider Figure 1. The line-of-sight (LOS) between the robot at  $(x_r, y_r)$  and the target at  $(x_t, y_t)$  is characterized by

$$V_R = \dot{R} = v_2 \cos(\alpha_2 - \theta) - v_1 \cos(\alpha_1 - \theta) \quad (4)$$

$$V_\theta = R\dot{\theta} = v_2 \sin(\alpha_2 - \theta) - v_1 \sin(\alpha_1 - \theta) \quad (5)$$

where  $V_R$  is the relative velocity component with respect to the robot along the LOS and  $V_\theta$  is the relative velocity component with respect to the robot perpendicular to the LOS. As shown in Figure (1), we assume

$$\omega_1 = \alpha_1 - \theta \quad (6)$$

$$\omega_2 = \alpha_2 - \theta \quad (7)$$

on differentiating above equations, we have

$$\dot{\omega}_1 = \dot{\alpha}_1 - \dot{\theta} \quad (8)$$

$$\dot{\omega}_2 = \dot{\alpha}_2 - \dot{\theta} \quad (9)$$

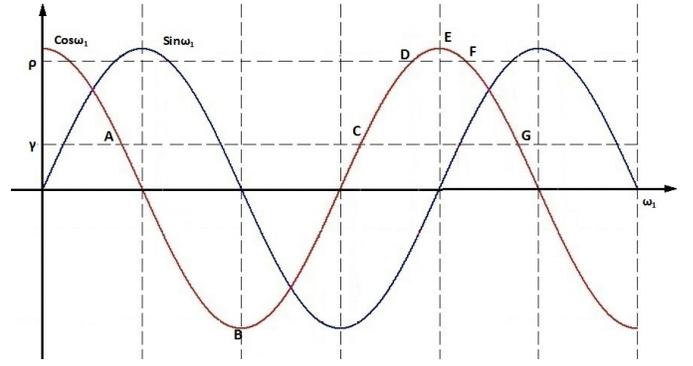


Fig. 2.  $\cos \omega_1$  and  $\sin \omega_1$  v/s  $\omega_1$  plots

Now let us say

$$\eta = \frac{v_2}{v_1} \quad (10)$$

From equation (4), (5), (6), (7) and (10), we have

$$V_R = \dot{R} = v_1(\eta \cos \omega_2 - \cos \omega_1) \quad (11)$$

$$V_\theta = R\dot{\theta} = v_1(\eta \sin \omega_2 - \sin \omega_1) \quad (12)$$

We assume that  $\eta \in [0, \gamma]$ , where  $\gamma$  is max velocity ratio till which robot can capture the target. We find the value of  $\gamma$ .

Now, from (11), we observe that, if  $\cos \omega_1 > \gamma$ , then we will have  $V_R < 0$ . Plots of  $\cos \omega_1$  and  $\sin \omega_1$  with respect to  $\omega_1$  is shown in Figure 2. Now consider a region where  $\rho \leq \cos \omega_1 \leq 1$  where  $\rho > \gamma$ . We will use  $\rho$  in defining the control law. From (11), we can find the bounds on  $V_R$  as  $V_R \in [-(\gamma + 1)v_1, (\gamma - \rho)v_1]$ . For the region  $\cos \omega_1 \leq \gamma$ , the bound will be  $V_R \in [-2\gamma v_1, (\gamma + 1)v_1]$ . We choose the value of  $\rho$  such that these bounds on  $V_R$  do not overlap, that is,

$$(\gamma - \rho)v_1 < -2\gamma v_1 \quad (13)$$

on simplifying we get  $\rho > 3\gamma$ . Since  $\rho \leq 1$ , we have  $3\gamma < 1$ , which implies

$$\gamma < \frac{1}{3}$$

Next, we find the control law that ensures that the target can be captured.

*Theorem 1:* A robot with dynamics given in (1)-(3) will be able to capture the moving target from any initial condition if it uses a control law given by

$$u = f(V_R, R) \quad (14)$$

where  $f(V_R, R)$  is define as

$$f(V_R, R) \in \begin{cases} \left( -\infty, \frac{-(\gamma + \sqrt{1 - \rho^2})v_1}{R} \right], & V_R \leq (\gamma - \rho)v_1; \\ (\dot{\alpha}_{1 \max}, \dot{\alpha}_{1 \min}), & V_R \in ((\gamma - \rho)v_1, -2\gamma v_1); \\ \left[ \frac{v_1(1 + \gamma)}{R}, \infty \right), & V_R \geq -2\gamma v_1; \end{cases} \quad (15)$$

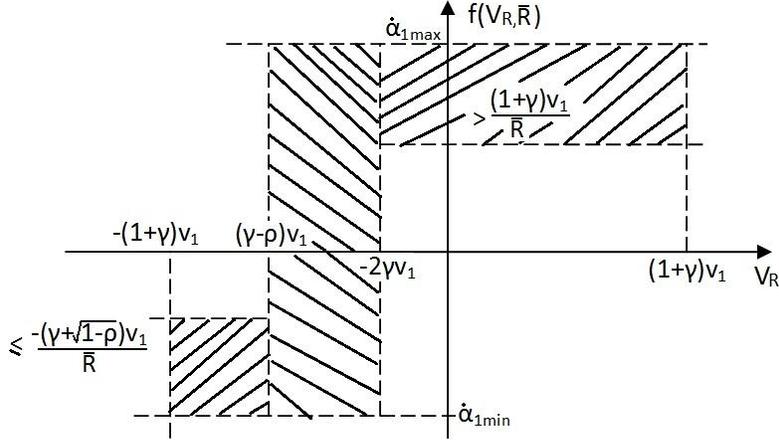


Fig. 3.  $f(V_R, \bar{R})$  v/s  $V_R$  plot

*Proof:* To capture the target we need  $V_R < 0$  which can be assured if  $\cos \omega_1 > \gamma$  for all time  $t > 0$ . So we will analyze how  $\cos \omega_1$  is varying with time. Let

$$z = \cos \omega_1 \quad (16)$$

on differentiating we get

$$\dot{z} = -\dot{\omega}_1 \sin \omega_1 \quad (17)$$

From Equations (3), (8), (12) and (14), we have

$$\dot{z} = -\sin \omega_1 \left( f(V_R, R) - \frac{V_\theta}{R} \right) \quad (18)$$

Since  $V_R$  is bounded as  $V_R \in [(\gamma+1)v_1, -(\gamma+1)v_1]$ , let us define

$$S_1 = \{V_R | -(1+\gamma)v_1 \leq V_R \leq (\gamma-\rho)v_1\} \quad (19)$$

$$S_2 = \{V_R | (\gamma-\rho)v_1 < V_R < -2\gamma v_1\} \quad (20)$$

$$S_3 = \{V_R | -2\gamma v_1 \leq V_R \leq (1+\gamma)v_1\} \quad (21)$$

For a particular value of  $R = \bar{R}$ ,  $f(V_R, \bar{R})$  lies in shaded region shown in Figure 3.

With reference to Figure 2, we consider the following cases.

**Case 1:** Between A and B, where  $-1 < \cos \omega_1 \leq \gamma$  and  $\sin \omega_1 > 0$ . In this region, we have from (11) and (12),  $V_R \in [-(\eta+\gamma)v_1, (\eta+1)v_1] \subseteq S_3$  and  $V_\theta \in [-(1+\eta)v_1, \eta v_1]$ . This implies from (15) that  $f(V_R, R) > \frac{v_1(\gamma+1)}{R}$ . We know that

$$\frac{(1+\gamma)v_1}{R} > \frac{\eta v_1}{R} \quad (22)$$

so, from Equations (18) and (22), we observe that  $\dot{z} < 0$  for any value of  $\eta \in [0, \gamma]$ . It implies that any point between A and B on  $\cos \omega_1$  curve will move towards B.

**Case 2:** Between B and C where  $-1 < \cos \omega_1 \leq \gamma$  and  $\sin \omega_1 < 0$ . Similar to the previous case, in this region  $V_R \in [-(\eta+\gamma)v_1, (\eta+1)v_1] \subseteq S_3$  and  $V_\theta \in (-\eta v_1, (1+\eta)v_1]$ . This implies  $f(V_R, R) > \frac{v_1(\gamma+1)}{R}$ . We know that

$$\frac{(1+\gamma)v_1}{R} \geq \frac{(1+\eta)v_1}{R} \quad (23)$$

so from equation (18) and (23), we observe that  $\dot{z} > 0$  for any value of  $\eta \in [0, \gamma]$ . This implies that any point between B and C on  $\cos \omega_1$  curve will move towards C.

**Case 3:** At B where  $\cos \omega_1 = -1$  and  $\sin \omega_1 = 0$ . At this point  $\dot{z} = 0$ , so we can not comment on the direction of movement of the point on the  $\cos \omega_1$  curve. Let

$$y = \sin \omega_1 \quad (24)$$

on differentiating

$$\dot{y} = \dot{\omega}_1 \cos \omega_1 \quad (25)$$

from Equations (3), (8), (12) and (14),

$$\dot{y} = \cos \omega_1 \left\{ f(V_R, R) - \frac{V_\theta}{R} \right\} \quad (26)$$

At B,  $V_R \in S_3$ ,  $V_\theta \in [-\eta v_1, \eta v_1]$  and  $f(V_R, R) > \frac{(\gamma+1)v_1}{R} > \frac{\eta v_1}{R}$ . Therefore,  $\dot{y} < 0$  which implies that at B the point will move towards C.

**Case 4:** Between D and E, where  $\rho \leq \cos \omega_1 < 1$  and  $-\sqrt{1-\rho^2} \leq \sin \omega_1 < 0$ . In this region,  $V_R \in S_1$ ,  $V_\theta \in (-\eta v_1, (\eta + \sqrt{1-\rho^2})v_1]$  and  $f(V_R, R) \leq \frac{-(\gamma + \sqrt{1-\rho^2})v_1}{R}$ . We know

$$\frac{-(\gamma + \sqrt{1-\rho^2})v_1}{R} < \frac{-\eta v_1}{R} \quad (27)$$

so from Equations (18) and (27), we observe that  $\dot{z} < 0$  for any value of  $\eta \in [0, \gamma]$ . This implies that any point in the region will move towards point D.

**Case 5:** Between E and F, where  $\rho \leq \cos \omega_1 < 1$  and  $0 < \sin \omega_1 \leq \sqrt{1-\rho^2}$ . In this region,  $V_R \in S_1$ ,  $V_\theta \in [-(\eta + \sqrt{1-\rho^2})v_1, \eta v_1]$  and  $f(V_R, R) \leq \frac{-(\gamma + \sqrt{1-\rho^2})v_1}{R}$ . Again,

$$\frac{-(\gamma + \sqrt{1-\rho^2})v_1}{R} \leq \frac{-(\eta + \sqrt{1-\rho^2})v_1}{R} \quad (28)$$

so from Equations (18) and (28) we observe that  $\dot{z} \geq 0$  implies that at any point in the region will move towards E.

**Case 6:** At E where  $\cos \omega_1 = 1$  and  $\sin \omega_1 = 0$ . Here  $\dot{z} = 0$ , so we will use (26). In this region,  $V_R \in S_1$ ,

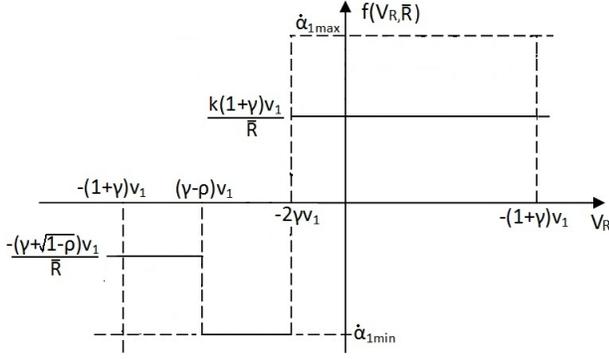


Fig. 4. A candidate Switching control input function

$V_\theta \in [-v_1\eta, v_1\eta]$  and

$$f(V_R, R) \leq \frac{-(\gamma + \sqrt{1-\rho^2})v_1}{R} < \frac{-v_1\eta}{R} \quad (29)$$

So from Equation (26), we can observe that  $\dot{y} < 0$  which implies that at E the point will move towards D.

Points between C and D will remain within C and D irrespective of the behavior of the control law in this region. Similarly, the points between F and G can i) remain within F and G, ii) move towards F and cross it or iii) move towards G and cross it. When a point crosses beyond G, it will fall under Case 1.

Hence we can conclude that any point will always end up in the region where  $\cos\omega_1 > \gamma$  which implies  $V_R < 0$  for all time and hence capture is inevitable. ■

Next we present some candidate control laws.

#### A. Switching Control

We propose a switching control law given as

$$f(V_R, R) = \begin{cases} \frac{-(\gamma + \sqrt{1-\rho^2})v_1}{R}, & V_R \in S_1; \\ a, & V_R \in S_2; \\ \frac{k_2 v_1 (1+\gamma)}{R}, & V_R \in S_3; \end{cases} \quad (30)$$

where  $a \in [\dot{\alpha}_{1\min}, \dot{\alpha}_{1\max}]$  and  $k > 1$ . This control function is shown in Figure 4.

#### B. Piecewise Continuous Control

We propose a piecewise continuous control law given as

$$f(V_R, R) = \begin{cases} \frac{-k_1(\gamma + \sqrt{1-\rho^2})v_1}{R}, & V_R \in S_1; \\ mV_R + c, & V_R \in S_2; \\ \frac{k_2 v_1 (1+\gamma)}{R}, & V_R \in S_3; \end{cases} \quad (31)$$

where  $m$  and  $c$  are

$$m = \frac{k_2(1+\gamma) + k_1(\gamma + \sqrt{1-\rho^2})}{R(-3\gamma + \rho)} \quad (32)$$

$$c = \frac{k_2(1+\gamma)v_1}{R} + m(2\gamma v_1) \quad (33)$$

and the  $k_1, k_2$  are defined as

$$1 \leq k_1 \leq \frac{\dot{\alpha}_{1\min} R_c}{(\gamma + \sqrt{1-\rho^2})v_1} \quad (34)$$

$$1 < k_2 \leq \frac{\dot{\alpha}_{1\max} R_c}{(1+\gamma)v_1} \quad (35)$$

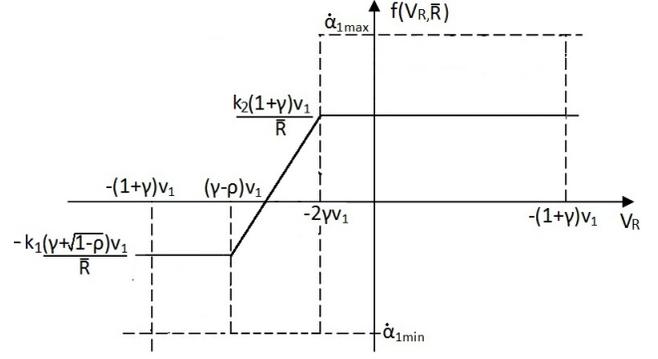


Fig. 5. A candidate Piecewise continuous control input function

so we chose the value of  $R_c$  such that it satisfies the equation (34) and (35). By properly defining the parameters in Equation (31), we can have piecewise continuous control law shown in Figure 5.

## IV. SIMULATION RESULTS

We considered a target moving A) in a straight line path and B) in a circular path. To capture the target, we applied both candidate control laws. For the simulation, we assumed  $v_1 = 10$ ,  $\gamma = 0.3$ ,  $\rho = 0.95$ ,  $\dot{\alpha}_{\max} = 5$ ,  $\dot{\alpha}_{\min} = -5$  and  $\eta = 0.25$ . The control laws will be as follows.

1) *Switching control*: Assuming  $a = \dot{\alpha}_{1\min}$  and  $k = 1.001$  and using Equation (30), we get

$$f(V_R, R) = \begin{cases} \frac{-6.1225}{R}, & V_R \in [-13, -6.5]; \\ -5, & V_R \in (-6.5, -6); \\ \frac{13.013}{R}, & V_R \in [-6, 13]; \end{cases} \quad (36)$$

For this control strategy, the capture radius is  $R_c \leq 0.26026$ .

2) *Piecewise continuous control*: Using Equation (32)-(35), we obtain  $m = \frac{3.2271}{R}$ ,  $c = \frac{32.3756}{R}$ ,  $k_1 = 1$ ,  $k_2 = 1.001$  and  $R_c = 2.6026$ . Then,

$$f(V_R, R) = \begin{cases} \frac{-6.5}{R}, & V_R \in [-13, -6.5]; \\ mV_R + c, & V_R \in (-6.5, -6); \\ \frac{13.013}{R}, & V_R \in [-6, 13]; \end{cases} \quad (37)$$

#### A. When target moves in straight path

The target is assumed at the different initial positions and  $\dot{\alpha}_2 = 0$ . The initial position of the robot is origin and the time taken to reach the target is tabulated in Table I. The initial conditions are selected such that each case corresponds to the different sections in Figure 2. The initial distance between the target and the robot is same for all the cases, which is equal to  $R_0 = 50$ . The variation of  $R$  with respect to time for all the cases are plotted in Figure 6 and 7 for the switching control and piecewise continuous control respectively. Similarly, in Table I,  $t_{sc}$  and  $t_{pc}$  corresponds to the time taken to capture the target for switching control and piecewise control strategy respectively. It is observed that the time taken by both the strategies are comparable. The last two columns of the table refers to the section in Figure 2, which highlights that the time taken can be significantly different when the initial condition is in this section.

Cases	1 (A)-(B)	2 at (B)	3 (B)-(C)	4 (C)-(D)	5 (D)-(E)	6 at (E)	7 (E)-(F)	8 (F)-(G)	9 (F)-(G)
$x_{t_0}$	40	9.95	-30	20	30	-40	33.01	-40	-40
$y_{t_0}$	-30	-49	40	45.83	40	-30	37.56	30	30
$\alpha_1$	$50^\circ$	101.4785	$25^\circ$	$25^\circ$	$50^\circ$	216.87	$45^\circ$	$165^\circ$	$210^\circ$
$\alpha_2$	$130^\circ$	$45^\circ$	$125^\circ$	$-45^\circ$	$145^\circ$	$15^\circ$	$30^\circ$	$15^\circ$	$15^\circ$
$V_{R_0}$	-2.981	8.619	4.55	-8.41	-10.0666	-12.32	-7.611	-10.824	-5.472
$V_{\theta_0}$	-9.417	2.084	9.71	4.29	3.0447	0.9312	-0.157	-5.691	-11.162
$\cos \omega_1$	0.0546	-1	-0.2057	0.749	0.9985	1	0.9979	0.928	0.393
$\sin \omega_1$	0.9985	0	-0.9786	-0.661	-0.0546	0	-0.0643	0.3725	0.919
$t_{sc}$	187.93	207.85	17.993	7.69	6.803	5.634	7.427	4.877	76.924
$t_{pc}$	185.30	203.09	17.742	7.54	6.688	5.587	7.2025	4.877	74.379

TABLE I

INITIAL CONDITIONS AND TIME TO CAPTURE FOR SWITCHING AND PIECEWISE CONTINUOUS CONTROL INPUT

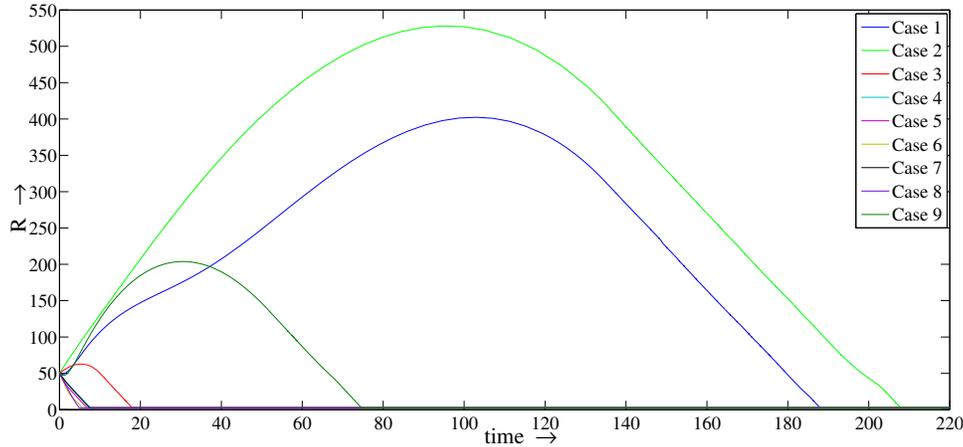


Fig. 6. R v/s time for all cases of Switching control when target moves in straight line

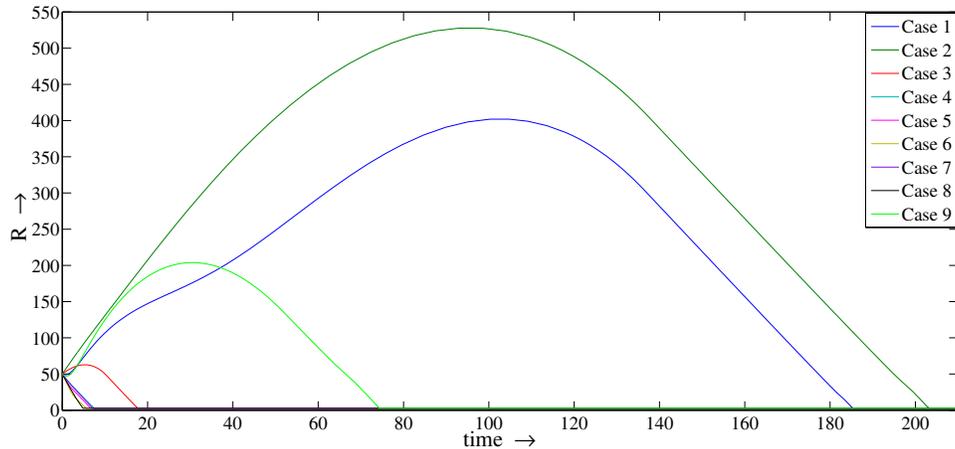


Fig. 7. R v/s time for all cases of Piecewise continuous control when target moves in straight line

### B. When target moves in circular path

In this case, we consider a particular initial condition given by  $x_{r_0} = 0, y_{r_0} = 0, \alpha_{1_0} = 210^\circ, \dot{\alpha}_1 = u, x_{t_0} = -40, y_{t_0} = 30, \alpha_{2_0} = 15^\circ, \dot{\alpha}_2 = 0.04$ . The initial  $\cos \omega_1$  point lies in the region between point *F* and *G* as shown in Figure 2.

For switching function the trajectory of the robot is shown in Figure (8). It is observed that the target has been captured. The control input is plotted in Figure 9, where we can see rapid switching of control law towards the end of the

trajectory.

Similarly for piecewise continuous control input, the trajectory of the robot is shown in Figure 10. It is observed that the target has been captured. The control input is plotted in Figure 11 and we observe that there is no switching in this case as expected.

### V. CONCLUSION

In the paper, we presented a strategy that can bring a robot arbitrarily close to a target point when the robot

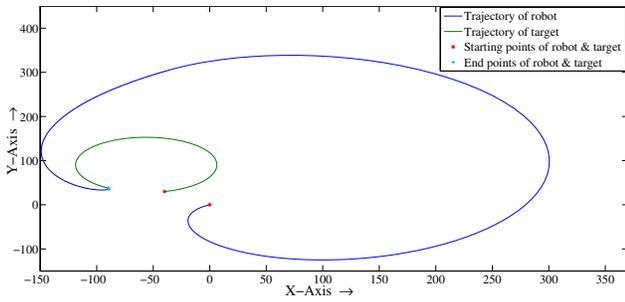


Fig. 8. Trajectory of robot and target for Switching control when target moves in circular path

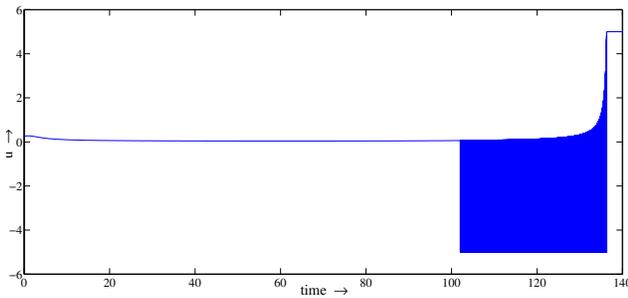


Fig. 9. Latax v/s time for Switching control when target moves in circular path

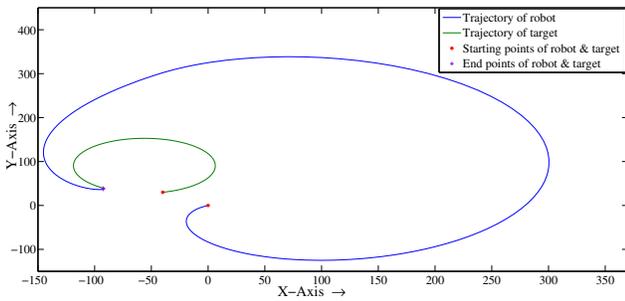


Fig. 10. Trajectory of robot and target for Piecewise continuous control when target moves in circular path

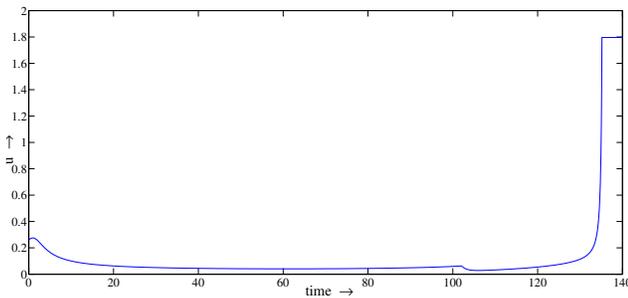


Fig. 11. Latax v/s time for Piecewise continuous control when target moves in circular path

can only measure the range and the range rate but has no information about the bearing angle to the target. We have derived the conditions that the control law should satisfy. Two sample functions are presented. When the robot is using these functions as the control law, it is shown in simulation that the robot can capture the target, thus validating our analysis. Since the measurement of range and range rate often include noise, an extension of this work will be to study the effect of noise on the control strategy. Implementation of the control law in a mobile robot or an UAV is the ultimate aim of this work.

## REFERENCES

- [1] Matveev AS, Teimoori H, Savkin AV. "The problem of target tracking based on range-only measurements for car-like robots," *Proceedings of the 48th IEEE Conference on Decision and Control*, Shanghai, China, 2009; pp. 85378542.
- [2] Teimoori H, Savkin AV. "Equiangular navigation and guidance of a wheeled mobile robot based on range-only measurements," *Robotics and Autonomous Systems* Feb. 2010; 58(2):203215.
- [3] Omid Namaki-Shoushtari, A. Pedro Aguiar, and Ali Khaki-Sedigh, "Target tracking of autonomous robotic vehicles using range-only measurements: a switched logic-based control strategy", in *International Journal of Robust and Nonlinear Control*, 2011, Published online in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/rnc.1806
- [4] S. D. Bopardikar, F. Bullo, and J. P. Hespanha. "A pursuit game with range only measurements". In *IEEE Conf. on Decision and Control*, pages 4233-4238, Cancun, Mexico, December 2008.
- [5] Branko Ristic, Sanjeev Arulampalam, and James hlcCarthy, "Target motion analysis using range-only measurements: Algorithms, performance and application" *ISAR data, Signal Processing*, vol. 82, pp. 273-296, Oct. 2001.
- [6] T.L. Song, "Observability of target tracking with range-only measurements," *IEEE Journal of Oceanic Engineering*, vol. 24, no. 3, pp. 383-387, July 1999.
- [7] Bingyi Fang, Siliang Wu and Guohua Weit. "Vector Miss Distance Measurement Based on Range-only Target Tracking," *International Conference on Radar*, 2006. CIE '06.
- [8] A. Gadre and D. J. Stilwell, "Toward underwater navigation based on range measurements from a single location," in *Proceedings of IEEE International Conference on Robotics and Automation*, pp. 44724477, 2004.
- [9] Mazomenos, E.B. and Reeve, J.S. and White, N.M., "racking with range-only measurements using a network of wireless sensors," *Sixth International Conference on Broadband Communications, Networks, and Systems*, pp. 1-8, Madrid, Spain, Sept. 2009.
- [10] Huang, G.P. and Zhou, K.X. and Trawny, N. and Roumeliotis, S.I., "A bank of maximum a posteriori estimators for single-sensor range-only target tracking", *Proceedings of the American Control Conference*, pp. 6974-6980, Baltimore, MD, USA, July 2010.
- [11] Fang, B. and Wu, S., "Angle-Parameterizations Range-only Target Tracking for Scalar Miss Distance Measurement System", *Proceedings of the 8th International Conference on Signal Processing*, Guilin, China, Nov. 2006
- [12] Kameda, H. and Tsujimichi, S. and Kosuge, Y., "Target tracking under dense environments using range rate measurements", *Proceedings of the 37th SICE Annual Conference*, pp. 927-932, China, Jul. 1998.
- [13] Zhou, K.X. and Roumeliotis, S.I., "Optimal motion strategies for range-only distributed target tracking", *Proceedings of the American Control Conference*, pp. 5195-5200, Minnesota, USA, June 2006
- [14] Ke Zhou and Roumeliotis, S.I., "Optimal Motion Strategies for Range-Only Constrained Multisensor Target Tracking", *IEEE Transactions on Robotics*, Vol. 24, No. 5, pp. 1168-1185, Oct 2008.
- [15] Sandini, G. and Lucarini, G. and Varoli, M., "Gradient driven self-organizing systems", *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 429-432, Yokohama, Japan, July 1993.
- [16] Ishida, H. and Ushiku, T. and Toyama, S. and Taniguchi, H. and Moriizumi, T., "Mobile robot path planning using vision and olfaction to search for a gas source", *Proceedings of the IEEE Conference on Sensors*, pp. 1112-1115, Nov 2005.