Cooperative Target Capturing in Minimum Time

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***Abstract*—In this paper, we look into the problem of capturing a stationary target using multiple MAVs. The aim is to have an uniform formation of the MAVs around the target in minimum time. To achieve the formation about the target, we use a modified cyclic pursuit strategy. There are various parameters in this strategy that affects the time required by the MAVs to get into the formation about the target. We characterize the effects of these parameters using Monte-Carlo simulations since an analytical solution may not be possible due to the non-linearities in this system.**

I. Introduction

The cyclic pursuit strategy has been studied in depth by various researchers and has proven to be a highly effective strategy in case of co-operative operation of multiple agents ([1] to [4], [8] and [9]) . The elegance of this strategy lies in the way the global behaviour of the agents is achieved by implementing local control. This enables control with less information i.e. only local information. Although cyclic pursuit currently has applications in scenarios involving co-operative operation of robots in various military and civil contexts, interest in this problem was an effect of a problem posed, about the trajectories that three dogs would trace if they were at the three vertices of an equilateral triangle and started to pursuit with constant speed, by Edouard Lucas back in 1877 [1].

Marshall et. al [2] studied the different formations of the agents possible using the cyclic pursuit strategy. The authors considered a situation in which among a group of n agents every agent(i) pursues the next agent($i+1$ modulo n). The motion of the agents is constrained to unicycles with constant speed. They found that the equilibrium formations of agents for the cyclic pursuit strategy used are regular polygons. The conditions for stability of these formations were also studied.

Sinha [3] has generalized the cyclic pursuit strategy into linear basic cyclic pursuit, linear centroidal cyclic pursuit, linear generalized centroidal cyclic pursuit and non-linear basic cyclic pursuit. The author has characterized the stable and unstable behaviour of agents and also invariance properties under the linear cyclic pursuit law. The equilibrium formations of agents under the basic non-linear pursuit law was also dealt with in detail by the author. All the linear pursuit laws lead to rendezvous where as the non-linear pursuit law leads the agents to achieve formations in the shape of regular polygons at equilibrium.

In all the strategies mentioned above, there is no predefined target present. This has been looked into by Rattan and Ghosh [4]. The authors have proposed the Implicit Leader Cyclic Pursuit which is a modification of the non-linear basic cyclic pursuit law to achieve equilibrium formation with the target at the centre. The authors achieve this by making one of the agents follow a virtual point on the line joining the next agent and the target point, while the remaining agents follow the next agent with non linear basic cyclic pursuit kinematics. The problem of target capturing is also looked into by Kobayashi et al. [5]. The authors have divided the task of target capturing into two separate tasks namely, enclosing and grasping. The important feature of the proposed method is its robustness.

Lan et al. [6] have implemented distributed control to achieve target enclosing by unicycle type robots. The authors have used a strategy where the individual motion of the agents is independent of the remaining agents. Thus they were successful in making the agents near the target to form a circle around the target while the agents far from the target reach the target moving linearly. The authors have further eliminated non holonomic constraints by a co-ordinate transformation [7]. Kim and Sugie [8] have developed a modified cyclic pursuit strategy for target capture in 3D. The authors have ensured collision avoidance using memory less controllers.

Daingade and Sinha [9] have analysed the modified cyclic pursuit strategy considered in this paper analytically. The authors have derived equations for radius and inter agent spacing at equilibrium. The authors also have derived a necessary condition for the existence of equilibrium.

Time to target plays a crucial role, especially in military applications. This motivated us to look into this problem. The parameters affecting time to target are identified and simulations are run for a range of values of these parameters and the value which provides minimum time to target is obtained. Also the cyclic pursuit strategy is modified by making velocity proportional to distance from target and its effect on time to target is analysed.

This paper is organized as follows: Modelling of the system for fixed agent velocity and setting up of initial conditions for simulations is done in section II. Simulations carried out for various ρ are discussed in section III. The proportional velocity strategy is looked into in section IV. Conclusions and Future Work are provided in section V.

II. Modelling

Consider a set of n non-holonomic, homogeneous agents. Non-holonomic agents are agents that cannot change their direction of motion instantaneously. Homogeneous agents implies all agents have the same velocity and turn rate which stay constant with time. The non-linear basic cyclic pursuit law dictates that the agent i follow the agent $i+1$ modulo n with the following kinematics

$$\begin{aligned} \dot{x_i} &= V\cos\alpha_i \\ \dot{y_i} &= V\sin\alpha_i \\ \dot{\alpha_i} &= \frac{K\phi_i}{V} \end{aligned} \tag{1}$$

where (x_i, y_i) is the position, V is the velocity, α_i is the heading of the i^{th} agent and ϕ_i is as defined in figure 1, here P_i is the position of the i^{th} agent.

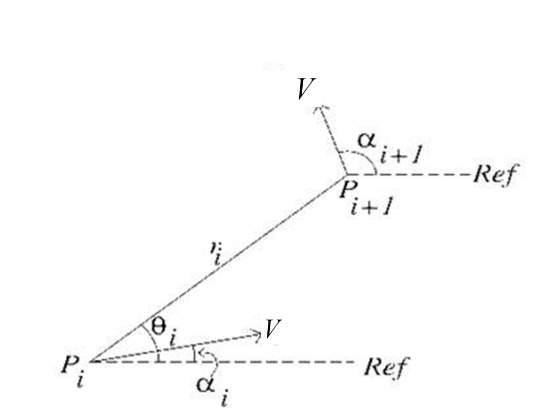

Fig. 1. Geometry of formation [3]

This pursuit law is modified [9] in order to achieve the final circular formation with the target at the centre. It can be seen from figure 1 that the various angles are defined with respect to the next agent. The modification is to replace the next agent by a virtual point (P') on the line joining the target and the next agent and all the angles are calculated with respect to this point. The location of P' on the line joining the target and the next line is decided by a factor ρ (figure 2). Rattan and Ghosh [4] introduce this modification for only one agent. This agent becomes the implicit leader and the remaining agents follow this agent with non linear basic cyclic pursuit. This strategy has been called 'Implicit Leader Cyclic Pursuit' [4]. Daingade and Sinha [9] introduce this modification for all the agents and we consider this case in this paper.

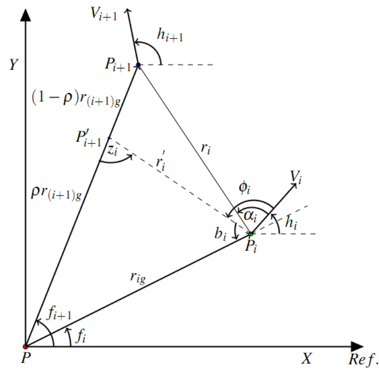

Fig. 2. Vehicle Formation geometry [9]

ρ is defined as the ratio in which the virtual point divides the line joining the target and the next agent. Since the agents follow the virtual point, the value of ρ effects the formation achieved at equilibrium. The extreme cases are when $\rho = 0$ i.e. the agents follow the target only and $\rho = 1$ when the agents follow only the next agent. When $\rho = 0$, the agents reach the the target quickly but fail to get into the equilibrium formation (regular polygon). When $\rho = 1$, the case boils down to the non-linear basic cyclic pursuit case without a target and the agents end up circling a point whose location is decided by the values of gain, velocity and initial conditions (positions and headings) of the agents.

The theoretical analysis for the above strategy has been done in [9] and is presented here for easy reference.

Theorem 1: At equilibrium, a system of n agents, with the kinematics (1), move around a stationary target with equal angular speed, maintaining constant distances between them ($\bar{r}$) and the target (R).

The equations are,

$$R = \frac{V^2}{K\phi_{eq}} \tag{2}$$

$$\bar{r} = \frac{V}{K\phi_{eq}}\sin(\pi\frac{d}{n}) \tag{3}$$

Where,

$$\phi_{eq} = \frac{\pi}{2} - \sin^{-1}\left(\frac{\rho\sin(2\pi\frac{d}{n})}{\sqrt{1+\rho^2-2\rho\cos(2\pi\frac{d}{n})}}\right) \tag{4}$$

In equations 3 and 4, d denotes the type of formation. It has been found that a system of n agents can reach equilibrium in $d = n-1$ number of formations. The different formations for $n = 5$ are shown in 3

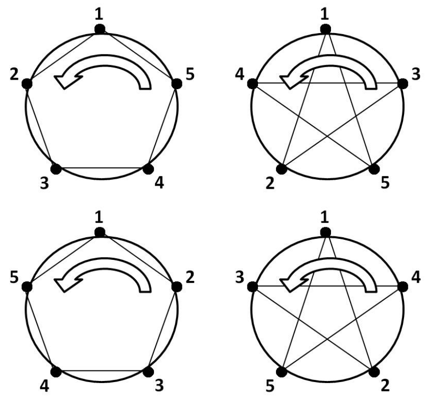

Fig. 3. Formations possible $d = 1to4$ (clockwise from top-left)

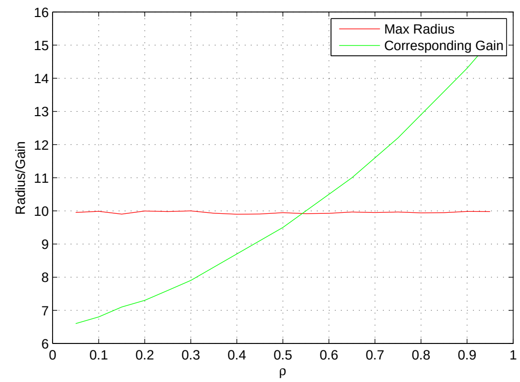

Fig. 4. Variation of K with ρ to maintain R_f constant

The objective is to find the time to reach target and select the parameters that minimize this time. The time to target is defined as the time at which the agents are both equally spaced from each other and are at the same distance from the target.

The time to target depends upon the initial positions and headings of the agents. In order to avoid influence of this factor on the simulation results the initial positions of the agents are restricted to a circle centred at the target and having a radius greater than the final equilibrium radius. The headings are always maintained towards the target and are randomized in an envelope of $\pm\ 45^0$ from the line joining the agent to the target.

The time to target also depends on the radius of formation at equilibrium. From equations 2 and 4 it can be seen that for a given K, V and ρ there are $n-1$ possible values of radius of formation R at equilibrium. The radius achieved is dependent on the type of formation d the agents get into. But there is no control over d, as it depends on initial positions, headings and ρ and is difficult to characterize. Hence there is a need to control the final radius of formation in some way to the extent possible. This problem is addressed by using different gain K for different ρ values. A desired maximum equilibrium radius of formation (R_f) is selected. This value is substituted in equation 2. In this equation the value of V is fixed. Therefore by substituting for different values of ρ we can calculate K which gives the desired R_f. Therefore the maximum radius is maintained the same for all the different values of ρ. This can be seen in figure 4 where the $R_i = 99$, $R_f = 11$ and the gains K for different ρ to achieve this R_f is plotted.

Even before the simulations are run the effect of ρ on the pursuit can be understood intuitively. Value of ρ ranges between 0 and 1. Consider the extreme cases.
When $\rho = 0$, the location of P' is the same as the location of the target and therefore all the agents pursue the target only and not the next agent. This leads to the agents reaching the target quickly (figure 5) but the agents will never be equally spaced (figure 6)

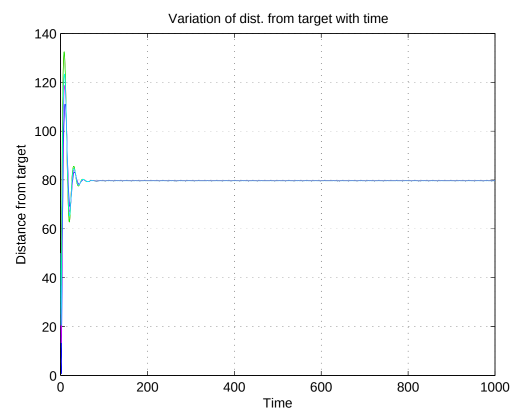

Fig. 5. Distance from target, $\rho = 0$

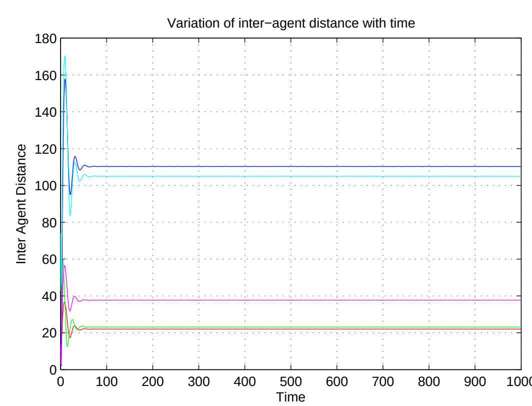

Fig. 6. Inter-Agent Distance, $\rho = 0$

When $\rho = 1$, the location of P' is the same as the location of the next agent and therefore all the agents pursue the next agent and not the target, resulting in non linear basic cyclic pursuit. Therefore the agents reach an arbitrary point depending on the initial conditions, K and V but not the target (figure 7) but the agents will be equally spaced (figure 8).

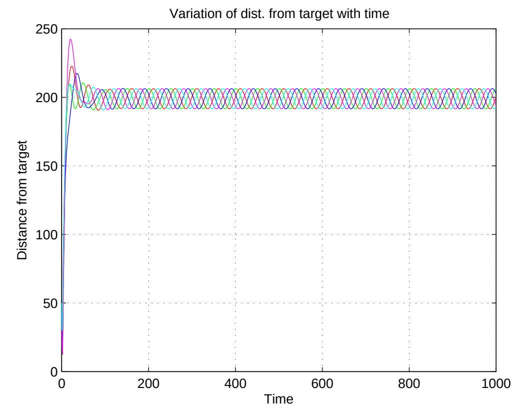

Fig. 7. Distance from target, $\rho = 1$

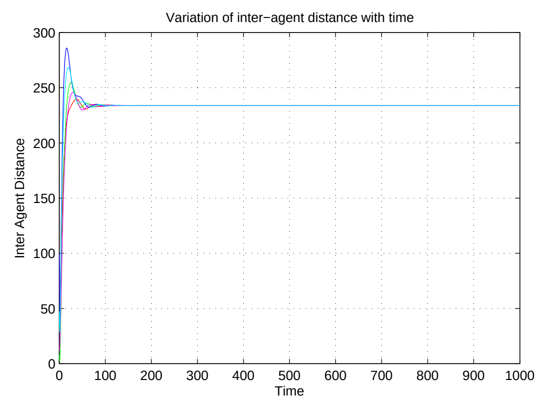

Fig. 8. Inter-Agent Distance, $\rho = 1$

III. Simulations

The agents can either be randomly located on the initial circle (R_i) with fixed headings or they can have fixed positions on the circle with random headings. The initial conditions need to be selected in order to account for randomness in both position and heading. Therefore the initial conditions used have both random positions and headings (figure 9). The radius of the circle along whose circumference the agents are placed is 50 and the target point is at the centre of this circle at (0,0). All the simulations carried are for five agents.

Simulations are run for different values of the ratio of radius of initial circle (R_i) to the maximum radius of equilibrium formation possible (R_f), that is $\frac{R_i}{R_f}$. The individual values of R_i and R_f are changed while the ratio is kept constant i.e. for $\frac{R_i}{R_f} = 10$ the simulation is done for $\frac{100}{10}, \frac{90}{9}, \frac{80}{8}, \frac{70}{7}$ and $\frac{60}{6}$. For every case in every ratio simulation is run for 100 random initial conditions for values of ρ between 0.05 and

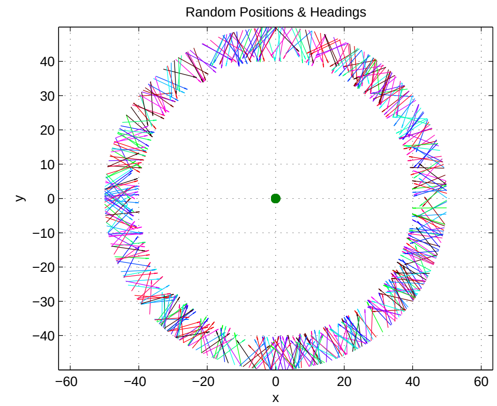

Fig. 9. Random positions and headings

0.95. The ρ value which provides minimum time to target is noted down. Five sets of R_i and R_f are considered for one value of the ratio $\frac{R_i}{R_f}$ as mentioned earlier. Therefore finally the data available is five values of $\rho_{t_{min}}$ for one value of $\frac{R_i}{R_f}$ (figure 10).

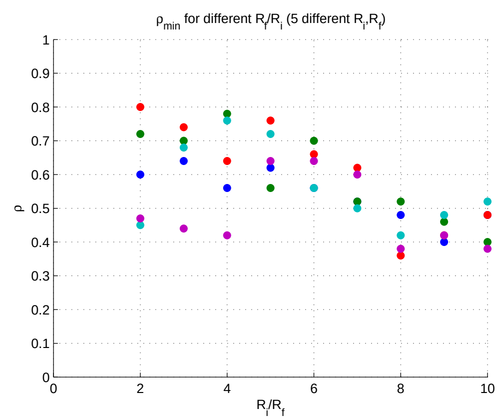

Fig. 10. ρ for minimum time to target

It can be observed (fig 10) that there is a general trend that for minimum time to target, $\rho \leq 0.5$ for $\frac{R_i}{R_f} = 8to10$ and $\rho \geq 0.5$ for $\frac{R_i}{R_f} = 1to7$.

Although the values of ρ which gives minimum time to target are obtained the study is not complete without the knowledge of occurrence of the different possible types of formation. In order to understand the variation of type of formation with ρ and $\frac{R_i}{R_f}$, the probabilities of occurrence of the different types of formation are obtained for 100 random initial conditions and are plotted in figures 11 through 19.

It is clearly observed that the formation $d = 2$ has the high probability of occurrence across all values of ρ, but the maximum probability is at $\rho = 0.5$ for all $\frac{R_i}{R_f}$. It is also observed that the probability of $d = 1$ is high when $\rho \geq 0.5$

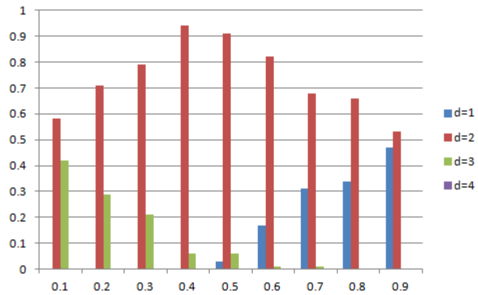

Fig. 11. $\frac{R_i}{R_f} = 2$

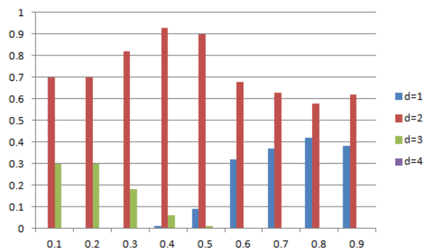

Fig. 12. $\frac{R_i}{R_f} = 3$

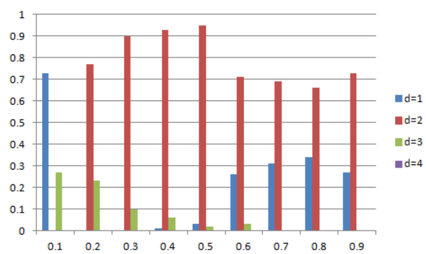

Fig. 13. $\frac{R_i}{R_f} = 4$

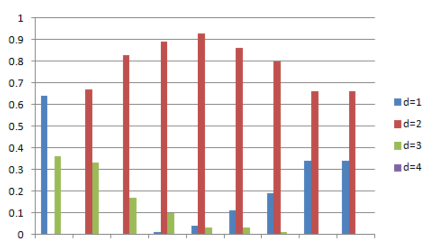

Fig. 14. $\frac{R_i}{R_f} = 5$

and the probability of $d = 3$ is high when $\rho \leq 0.5$. It is also important to note that $d = 4$ does not occur even once. However there are no theoretical studies as of now to support this observation.

IV. Proportional Velocity strategy

In this strategy the velocity of the agents is made proportional to their distance from the agent with constant of proportionality K_V. The kinematics of the agents is same as non-linear basic cyclic pursuit but only the control law is

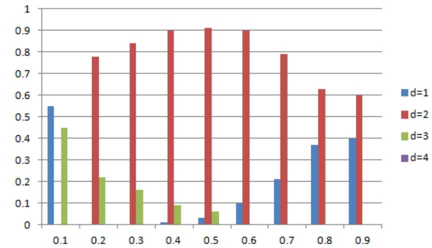

Fig. 15. $\frac{R_i}{R_f} = 6$

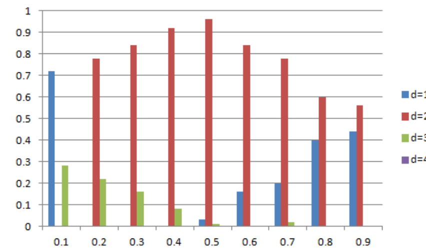

Fig. 16. $\frac{R_i}{R_f} = 7$

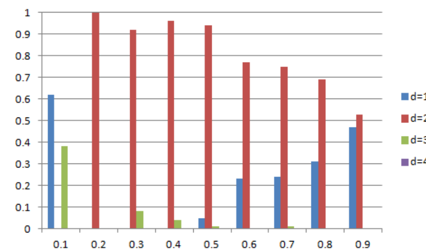

Fig. 17. $\frac{R_i}{R_f} = 8$

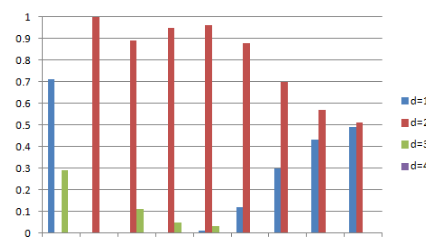

Fig. 18. $\frac{R_i}{R_f} = 9$

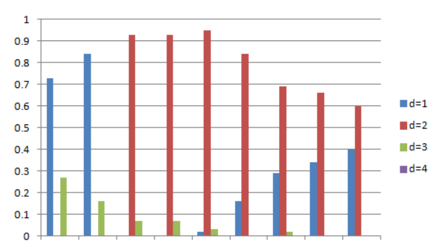

Fig. 19. $\frac{R_i}{R_f} = 10$

different and is given by

$$\begin{aligned} \dot{x}_i &= V\cos\alpha_i \\ \dot{y}_i &= V\sin\alpha_i \\ \dot{\alpha} &= K\phi_i \end{aligned} \tag{5}$$

Initially the simulations were carried out with the velocity term in the denominator of the $\dot{\alpha}$ equation. If no limits were imposed on the velocity the agents did not achieve pursuit and when limits were imposed pursuit was achieved but the time taken by the agents to reach target was very large. Therefore the velocity term was removed. Since the velocity is proportional to distance from the target, as the agents move closer to the target the velocity reduces and they end up meeting at the target point. To avoid this, velocity is given a lower limit. Also if the initial distance from the target is initially very high, then the agent will have a very high velocity and will take a long time to achieve cyclic pursuit, therefore the velocity is also given an upper limit.

To compare the effectiveness of this strategy with the fixed velocity non linear basic cyclic pursuit case, simulation is run for both the strategies with same initial conditions and target. The initial conditions are selected randomly in a 10 $\times$ 10 grid with (0,0) and (10,10) as two corners and the target is at (100,100). The simulations are carried out for 25 random initial conditions in the grid. The velocity for the fixed velocity case is 20 and the velocity bounds for the proportional velocity case are 15 and 25.The proportionality constant K_V = 2.5.

It is observed that the time taken reduces drastically. The proportional velocity strategy gives an average time to target of 12.51 seconds while the fixed velocity strategy gives an average time to target of 126.15 seconds. The time is reduced by a factor of around 10(Figure 20).

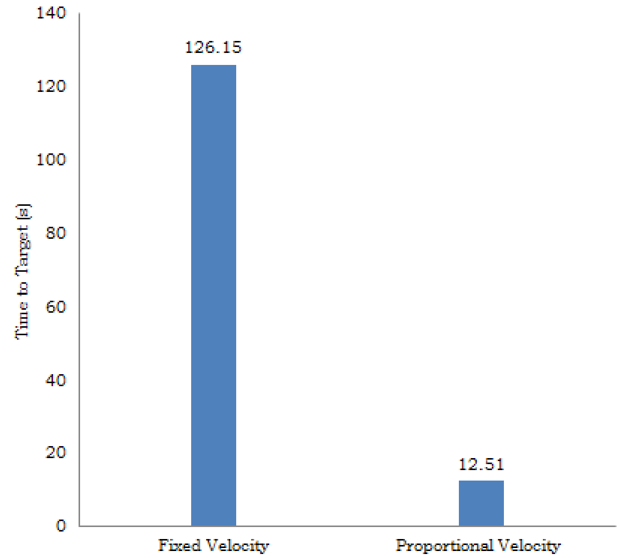

Fig. 20. Effectiveness of Proportional Velocity Strategy

V. Conclusion

In this paper we studied the simulation results and found that there is a general trend that for minimum time to target, $\rho \leq 0.5$ for $\frac{R_i}{R_f} = 8-10$ and $\rho \geq 0.5$ for $\frac{R_i}{R_f} = 1-7$.

The formation $d = 2$ has the high probability of occurrence across all values of ρ, but the maximum probability is at $\rho = 0.5$ for all $\frac{R_i}{R_f}$. It is also observed that the probability of $d = 1$ is high when $\rho \geq 0.5$ and the probability of $d = 3$ is high when $\rho \leq 0.5$. It is also important to note that $d = 4$ does not occur even once.

The proportional velocity strategy reduces the time to target drastically.

Future work on this problem is to find a method to fix the radius of formation at equilibrium and get a value of ρ for minimum time with certainty rather than depending on probabilities of occurrence of a type of formation.

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