Course: SC 309/201 - Mathematical structures for systems and control (3 0 0 6)

Venue: Individual residences (Remote mode), Time: Wed and Fri - 9.30 AM to 10.55 AM

Instructors: Ravi N. Banavar (Room No. 210, Ext.7888), Systems and Control (situated behind the Mathematics department.)

Prerequisites: None

Rough Outline:
The broad objective of the first part of this course is to introduce fundamental aspects of four topics in mathematics, that are highly useful in advanced engineering applications and studies.

- Vector spaces (5 weeks): Linear independence, basis, dimension, subspaces, linear functionals, dual spaces, linear transformations, inner products, normed vector spaces, eigen values and eigen vectors

- Sequences and series (2 weeks): Sequences, series and convergence, power series, differentiation and integration of a series

- Multivariable calculus (3 weeks): Taylor’s formula, differentiation of functions of several variables, Gâteaux and Frechet derivatives, differentiability of vector-valued functions, the chain rule, Taylor’s formula for functions of several variables

- Convexity and optimization (3 weeks): Unconstrained optimization, first-order and second-order optimality conditions, quadratic forms, affine geometry, convex sets, convex cones, convex functions, optimization on convex sets

References:

Evaluation scheme: (Not for sure, a tentative one): 3 to 4 quizzes (55 %), class participation (25 %), assignments (20 %).
Week-wise coverage:

1. Week 1 - Fields and vector spaces, linear independence, span, basis, dimension, subspace, linear functionals,

2. Week 2 The dual space, dimension of the dual space, annihilator of a subspace, linear transformations (homomorphisms),

3. Week 3 - Linear transformations, invertible linear transformations, matrix representation of a linear transformation, the adjoint transformation, similarity transformation, the projection transformation,

4. Week 4 - Norms and inner products, norm induced by the inner product Cauchy Schwarz inequality, Bessel’s inequality, the Reisz representation theorem

5. Week 5 - Eigen values and eigen vectors, geometric and algebraic multiplicity of eigen values

6. Week 6 - Series, Convergence, absolute convergence

7. Week 7 - Power series, differentiation and integration of a series, continuous maps, limits in function spaces

8. Week 8 - Differentiation of functions of several variables, Gâteaux and Fréchet differentiability, the chain rule

9. Week 9 - Inverse function and implicit function theorems

10. Week 10 - Optimization of smooth functions, local optimizers, Weierstrass’ theorem,

11. Week 11 - First order optimialty, second order optimality conditions, quadratic forms

12. Weeks 12 and 13 - Convex analysis, convex sets, convex functions