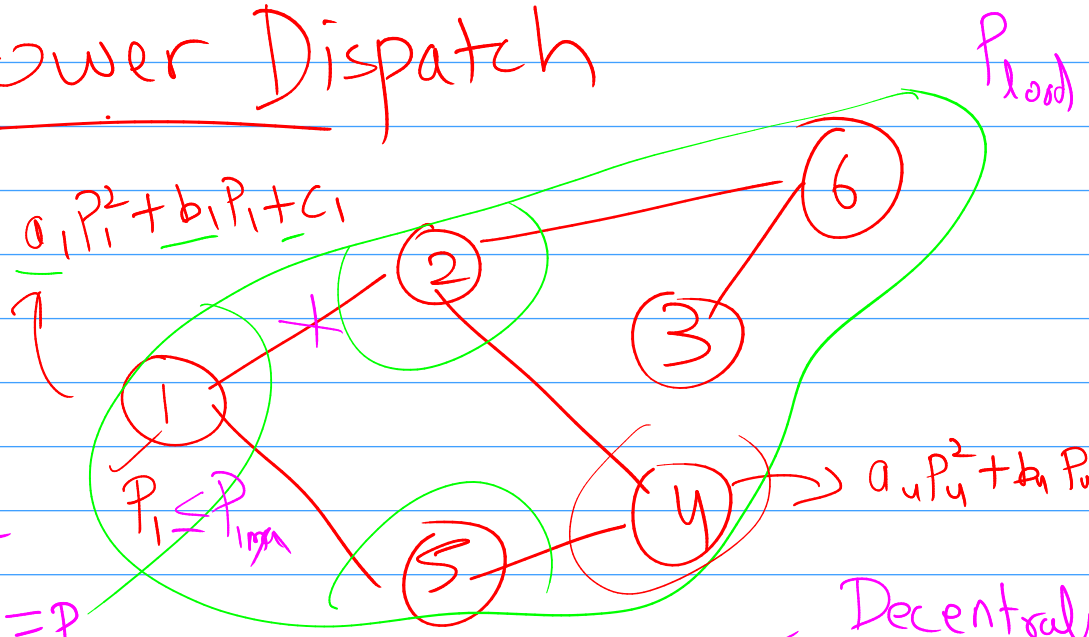
SC 646: Distributed Optimization & ML

Economic Power Dispatch



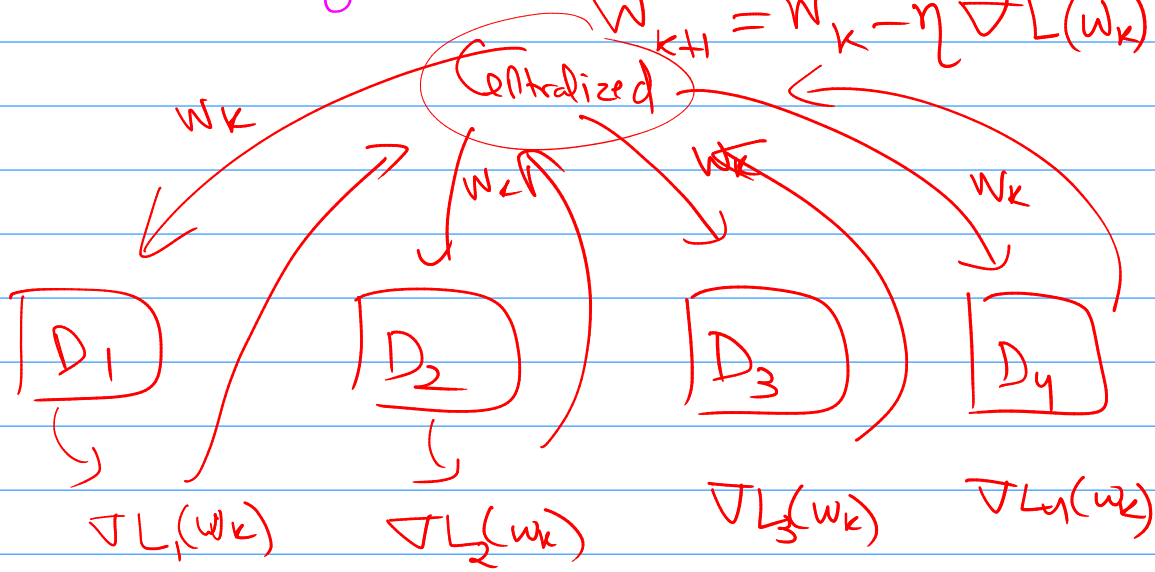
$P_1+P_2+P_3+$
$P_4+P_5+P_6=P_{load}$

Decentralized

$$\sum_{i=1}^{6} a_iP_i^2+b_iP_i+c_i \rightarrow$$

$$\text{s.t} \sum_{i=1}^{6} P_i = P_{load}$$

$$\underbrace{P_{i,min}}_{0} \le P_i \le P_{i,max}$$



$$w_{k+1} = w_k - \frac{\eta}{4}\left(\nabla L_1(w_k)+\nabla L_2(w_k)+\dots \nabla L_4(w_k)\right)$$

$$\min \sum_{i=1}^{4} (x_i - i)^2$$

$$\Leftrightarrow x_1 = x_2 = x_3 = x_4$$

Decentralized

$$\min \sum_{i=1}^{4} (x - i)^2$$

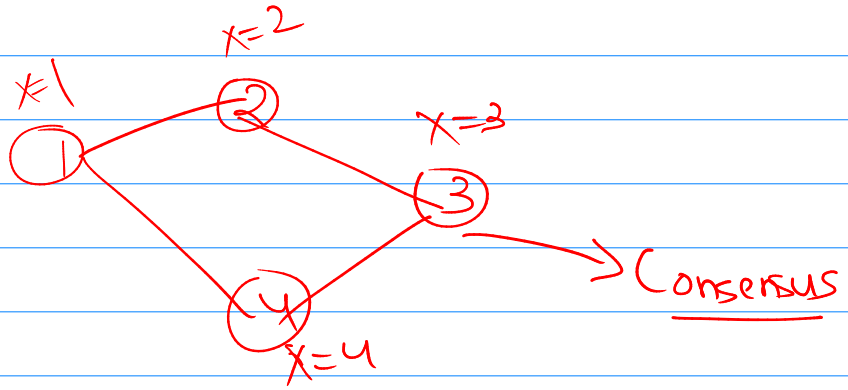
Centralized

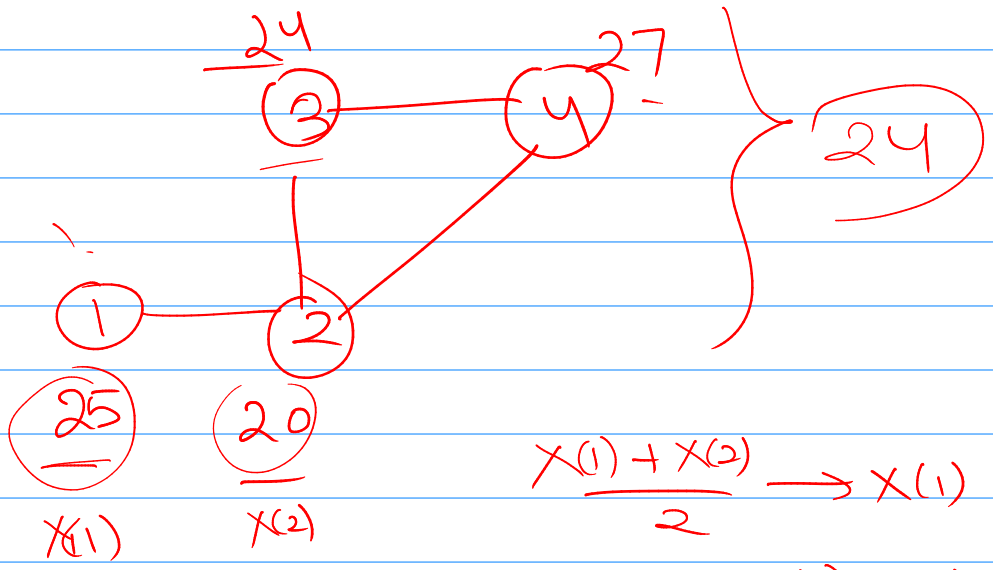
1, 2, 3, 4

$2.5 = x^*$

$(x_1 - 1)^2 \quad (x_2 - 2)^2$

$x_1^* = 1 \qquad x_2^* = 2$





$$\frac{X(1) + X(2)}{2} \rightarrow X(1)$$

$$\begin{bmatrix} X(1) \\ X(2) \\ X(3) \\ X(4) \end{bmatrix}_{k+1} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} X(1) \\ X(2) \\ X(3) \\ X(4) \end{bmatrix}_k \rightarrow A$$

$$\frac{X(1) + X(2) + X(3) + X(4)}{4} \rightarrow X(2)$$

$$\frac{X(2) + X(3) + X(4)}{3} \rightarrow X(3)$$

$$\frac{X(2) + X(3) + X(4)}{3} \rightarrow X(4)$$

$$\bar{X}_{k+1} = A \bar{X}_k$$

$$x_{k+1} = x_k - \eta \nabla F(x_k)$$

$\min_x F(x)$

↳ Gradient Descent (GD)

$$\frac{x_{k+1} - x_k}{\eta} = -\nabla F(x_k) \longrightarrow \dot{x} = -\nabla F(x)$$

$$\lim_{\eta \to 0} \left(\frac{x_{k+1} - x_k}{\eta} \right) = -\nabla F(x_k)$$

↳ Gradient Flow (GF)

$$F(x) = \frac{1}{2} x^2$$

$$\nabla F(x) = x$$

$$\dot{x} = -x$$

$x^* = 0$ Equilibrium Pt / Optimal soln

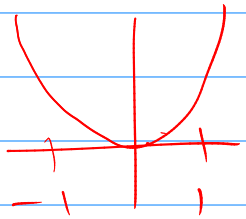
$$x_{k+1} = x_k - \eta \frac{\nabla F(x_k)}{\|\nabla F(x_k)\|}$$

$$x(t) = e^{-t}$$

→ Exponentially stable

$$\dot{x} = -\frac{\nabla F(x)}{\|\nabla F(x)\|}$$

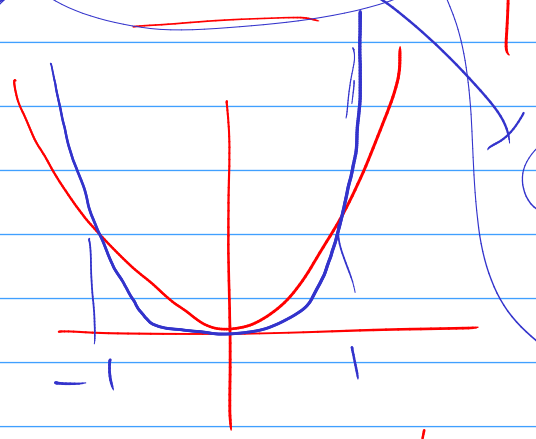
$$\dot{x} = -\text{sign}(x)$$



$\nabla F(x) = x$

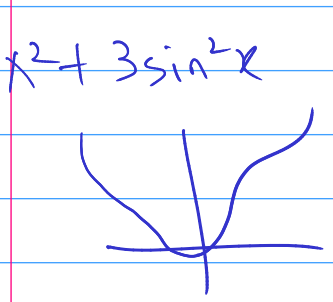
$$\dot{x} = -x$$

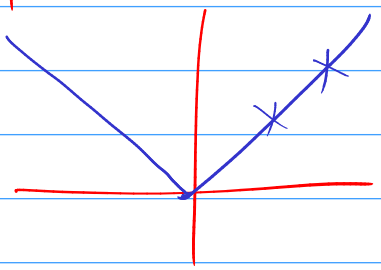
$F_1(x) = \frac{1}{2}x^2$ | $F_2(x) = \frac{1}{4}x^4$.



$x \gg x^3$

→ Strongly Convex Function





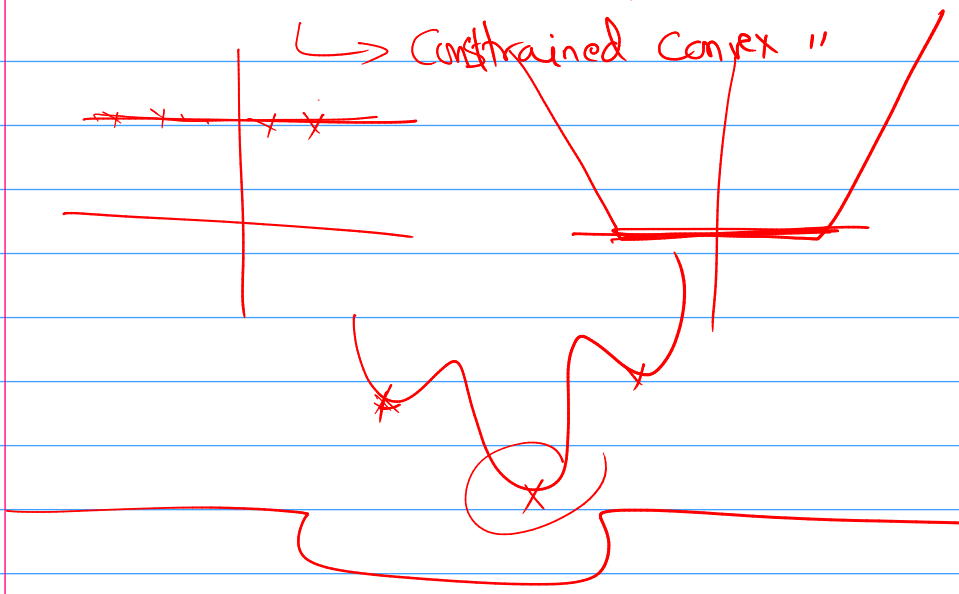
→ $F(x) = |x|$

PL inequality

Course Outline

↳ Convex optimization

↳ Constrained convex "



- Lyapunov stability Theory
  - ↳ Fixed-time stability Theory
- Graph Theory
- Distributed Optimization
- Federated Learning (Optimal)

→ 2-3 HWs (40%)

→ Midterm/Paper Presentations (20%)

→ Endterm (40%)