Homework Problems

1. Show that if $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are two wavefunctions of Schröedinger equation then

$$\frac{d}{dt}\int_{-\infty}^{\infty}\Psi_2(x,t)^*\Psi_1(x,t)=0.$$

2. A particle at $t=0$ is represented by the wavefunction

$$\psi(x) = A(a^2-x^2), \quad |x|<a \tag{1}$$
$$= 0, \quad \textit{otherwise} \tag{2}$$

   Find A. Find expected value of x, x^2, p, p^2 at $t=0$.

3. Recall we said our problem is quantum mechanical when Debroglie wavelength $\lambda=\frac{h}{p}$ is greater than characteristic length at which potential varies. In solid state potential due to atomic nuclei varies at scale of inter atomic spacing of say $3A^\circ$. Show that at room temperatures ($300K$ electrons in solids are quantum mechanical). You may estimate the velocity of the electron from $\frac{1}{2}mv^2=\frac{3}{2}kT$.

4. Let $\Psi(x,t)$ be solution to Schröedinger equation. Show that expected value $\langle E\rangle = \langle\Psi, H\Psi\rangle$ stays constant.

5. A particle in an infinite square potential as described has initial wavefunction

$$\Psi(x,0)=A\sin^3\frac{\pi x}{a}, \quad 0<x<a.$$

   Find A, $\Psi(x,t)$, $\langle x\rangle$ and $\langle E\rangle$.

6. Find eigen-energies for a half harmonic oscillator with potential

$$V(x)=\frac{1}{2}kx^2, \quad x\geq 0,$$

   and ∞ for $x<0$.

7. For the double well potential shown in figure, find the ground state and first excited state wavefunction ψ_0 and ψ_1 when $b=0$, $b\sim a$ and $b\gg a$. Plot the corresponding energies $E_0(b)$ and $E_1(b)$ as function of b.

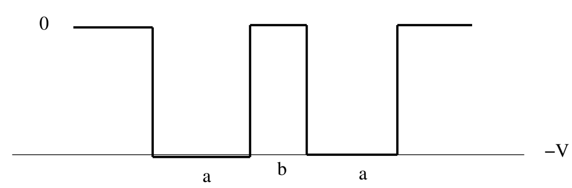

Figure 1: Fig. shows a double well potential.

8. Construct the wavefunction of hydrogen in state $n = 4, l = 3, m = 3$ and express ψ in terms of r, θ, ϕ. Find $\langle r \rangle$. If we measure $L_x^2 + L_y^2$ on this wavefunction, what values can be expect and with what probability.

9. Consider a simple cubic lattice with a and t as the lattice parameter and the hopping parameter and onsite energy ϵ_0. Find dispersion relation $\epsilon(k)$, where assume interaction with nearest nghbs. Assuming one electron per site, how is the band filled.

10. Consider a BCC lattice with a and t as the lattice parameter and the hopping parameter and onsite energy ϵ_0. Find dispersion relation $\epsilon(k)$. where assume interaction with nearest nghbs. Assuming one electron per site how is the band filled.

11. Consider a FCC lattice with a and t as the lattice parameter and the hopping parameter and onsite energy ϵ_0. Find dispersion relation $\epsilon(k)$, where assume interaction with nearest nghbs. Assuming one electron per site how is the band filled.

12. Consider a Silicon lattice with a and t as the lattice parameter and the hopping parameter and onsite energy ϵ_0. Assuming four sp^3 hybridized electron per site, find dispersion relation $\epsilon(k)$ for valence and conduction band.

13. Consider a Gallium-Arsenide lattice with a and t as the lattice parameter and the hopping parameter and onsite energy ϵ_1 on Gallium and ϵ_2 at Aresenic. Assuming four sp^3 hybridized electron per site find dispersion relation $\epsilon(k)$ for valence and conduction band.

14. Consider a 1D periodic potential with lattice parameter $a = 3A^\circ$ and t parameter at $5eV$. Calculate the fermi-velocity of a half filled band.

15. Now consider a 3D periodic potential with simple cubic lattice and lattice parameter $a = 3A^\circ$ and t parameter at $5eV$. Calculate the fermi-velocity for an electron in half filled band on Fermi sphere in direction $(1, 0, 0)$.

16. Consider a 1D periodic potential with electrons treated in free electron approximation. With potential
$$V = V_0 \cos^2(\frac{\pi x}{a}) \tag{3}$$
with $V_0 = 100V$ and lattice parameter $a = 3A^\circ$. Sketch first two energy bands and find the band gap.

17. In the above find the band gap between 2^{nd} and 3^{rd} band.

18. Consider a 3D periodic potential with electrons treated in free electron approximation. With potential
$$V = V_0 \cos^2(\frac{\pi x}{a}) \cos^2(\frac{\pi y}{a}) \cos^2(\frac{\pi z}{a}), \tag{4}$$
with $V_0 = 100V$ and lattice parameter $a = 3A^\circ$. find the band gap between first two energy levels.

19. In class we considered a molecular bond between two atomic orbitals each with energy ϵ and transfer element $-t$. This was a covalent bond. Generalize this to case when atomic orbitals have energy ϵ_A and ϵ_B respectively. what are the energies of two moelcular orbitals bonding and antibonding. This is called an ionic bond. Like table salt, NaCl.

20. We considered tight binding in monoatomic chain. We now consider diatomic chain
$$A - B - A - B - \cdots - A - B$$
where onsite energy at site A is ϵ_A and site B is ϵ_B and transfer element is $-t$. Find the dispersion relation and sketch it for this chain.

21. A silicon ingot is doped with 10^{16} arsenic atoms/cm^3. Find the carrier concentrations and the Fermi level at room temperature (300 K).

22. Calculate the inbuilt potential for a silicon pn junction with $N_A = 10^{18}/cm^3$ and $N_D = 10^{15}/cm^3$ at 300K.

23. In Fig. 5.5B of the book assume DC votage on the base of $1V$ (no ac voltage). Find the collector current if the $\beta = 100$ and $R_1 = 100\Omega$.

24. In above what should be R_2 for an amplifier gain to be 100.

25. In Fig. 5.6C of the book, $R_1 = 100\Omega$, what should be R_2 for an amplifier gain to be 100.

26. In Fig. 5.6D of the book, $R_1 = 100\Omega$, what should be R_2 for an amplifier gain to be 10.

27. Consider metal calcium or magnesium with two electrons it the outer shell. This says that the conduction band will be full. Why is it a metal then.

28. Consider a simple cubic lattice with m as the atomic mass and k as the spring constant and a as the lattice parameter. Assume interaction with the nearest and second nearest nghbs. Calculate the phonon spectrum for transverse and longitudinal phonons and sketch first Brillouin zone.

29. Consider a BCC lattice with m as atomic mass and k as spring constant and a as the lattice parameter. Assume interaction with nearest nghbs. Calculate the phonon spectrum and sketch first Brillouin zone.

30. Consider a FCC lattice with m as atomic mass and k as spring constant and a as the lattice parameter. Assume interaction with nearest nghbs. Calculate the phonon spectrum and sketch first Brillouin zone.

31. Consider a 3D periodic potential

$$V = V_0 \cos^2(\frac{\pi x}{a}) \cos^2(\frac{\pi y}{a}) \cos^2(\frac{\pi z}{a}), \tag{5}$$

with $V_0 = 10V$ and lattice parameter $a = 3A^\circ$ amd atomic mass of 20 protons. At temperature $T = 100$K, calculate the parameter c in the book and use it to estimate the relaxation time τ assuming debye frequency of $\omega_d = 10^{13}$ Hz and Fermi energy $E_F = 10$ eV.

32. Phonon heat capacity of a solid per unit volume at low temperature goes as CT^3 , find the parameter C.

33. Electron heat capacity of a solid per unit volume goes as CT , find the parameter C, asumming Fermi energy $E_F = 10$ eV.

34. Given electric field of 1 V/m and relaxation time of electrons $\tau = 10^{-14}$ sec. What is the drift velocity of the electrons.

35. Consider a 3D periodic potential

$$V = V_0 \cos^2(\frac{\pi x}{a}) \cos^2(\frac{\pi y}{a}) \cos^2(\frac{\pi z}{a}), \tag{6}$$

with $V_0 = 10V$ and lattice parameter $a = 3A^\circ$ amd atomic mass of 20 protons. Calculate the parameter c in the book and use it to find the binding energy per electron and superconducting gap assuming debye frequency of $\omega_d = 10^{13}$ Hz and Fermi energy $E_F = 10$ eV.

36. In the above problem if the hopping parameter $t = 5$ eV, calculate ω_F and binding energy and superconducting gap.

37. How will the superconducting gap change if the mass of the ions was doubled.

38. How will the superconducting gap change if the spring constant of the lattice was doubled.

39. How will the superconducting gap change if the lattice parameter a was doubled.

40. How will the superconducting gap change if the hopping parameter t was doubled.

41. What is minimum magnetic field allowed through a hole of radius 1 mm in a superconductor.

42. Find the paramagnetic susceptibility of conduction electrons with fermi energy at 10 eV and density $10^{28}/m^3$.

43. Find the dimagnetic susceptibility of conduction electrons with fermi energy at 10 eV and density $10^{28}/m^3$.

44. Neon (atomic number 10) is FCC lattice with lattice parameter $a = 4.46A^{\circ}$. Find the dimagnetic susceptibility.

45. In quantum hall effect what is the height of the plateaus in units of Ω.

46. At a magnetic field of 10 T, what is the energy spacing between the the Landau levels.

47. In De Haas-van Alphen effect the susceptibility goes through a period as magnetic field is increased from 2.08 T to 2.1 T. What is the extremal area.

48. In the X-ray diffraction experiment, the wavelength of the X-ray is $3A^{\circ}$ and lattice spacing $4A^{\circ}$. What is the smallest angle at which diffraction is seen.

49. In X-ray diffraction experiment, the wavelength of the X-ray is $3A^{\circ}$. If diffraction is seen at 5° and then 20° what is the lattice spacing.

50. In Neutron spectroscopy of phonons, incident neutrons at $50eV$ are scattered (deflected) by an angle 30° from their course. If scattered neutrons have energy $50.01eV$. Find the magnitude of phonon energy and momentum.

51. In the Arpes experiment, X rays of wavelength $30A^{\circ}$ are used. Ejected electrons are emitted at angle 45° to the vertical and azimuth 60°. If the energy of the ejected electron is $10eV$, find the energy momentum of the valence electron.

52. For electrons at room temperature, what is the tunelling probability through a barrier of height 5 V and width 1 nm.