Geometry of Collectives: Control, Dynamics and Reconstruction

P. S. Krishnaprasad
University of Maryland, College Park
Electrical and Computer Engineering Department &
Institute for Systems Research

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Geometric ideas enter the investigation of collective behavior from multiple vantage points: the structure of configuration space; the synthesis of control strategies; the role of symmetry and reduction in closed loop dynamics; and the analysis of empirical data from biology. In this lecture we will present an overview of recent progress in these directions. We will consider methods to assimilate sampled observations of predator-prey encounters and bird flocking events into generative models based on differential equations with inputs and outputs. The purpose of such assimilation is to evaluate hypotheses of interest, based on correlations, delays, and mechanisms of interaction between elementary units of the observed population. Initial ideas on the development of control strategies were strongly influenced by studies in the laboratory (with Cynthia Moss and her students), on the prey-capture behavior of echolocating bats. Strategies found in these studies serve as building blocks for rules of collective behavior. Analysis of trajectory data (provided by Andrea Cavagna) on large flocks of starlings demands efficient reconstruction techniques. Again the data on prey-capture behavior of bats is a testing ground for our methods of reconstruction. We will discuss some robotics experiments guided by these studies.


Eric Justh


Cynthia Moss


Timothy Horiuchi


Kaushik Ghose


Chen Chiu


Vishwa Reddy


Ermin Wei


Kevin Galloway


Matteo Mischiati


Biswadip Dey

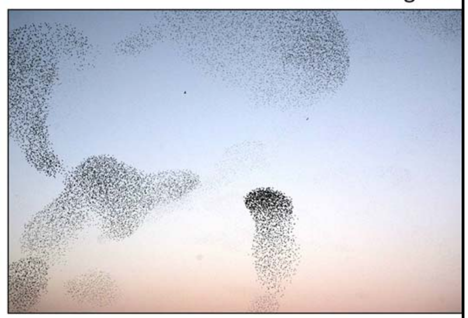

Andrea Cavagna (Co-PI)

COLLABORATORS

Views of collectives

Branta canadensis

Sturnus vulgaris



Starling Flock Movie – courtesy of Andrea Cavagna, Institute for Complex Systems, Rome.

Driving Questions about collectives

- What natural algorithms govern collectives?
- What building blocks (dyadic interactions)?
- What purpose?
- What collective strategies (from dyadic …)?
- How do we infer mechanisms from data?

Focus on animals with well-developed sensory modalities (vision, audition, olfaction, …), and well-developed freedom of movement in 3D

(animals with free will)

By natural algorithms we mean sensory-motor feedback laws that are involved in the behavior of natural collectives (flocks of birds, schools of fish, swarms of insects,…)

Dyadic interactions are pairwise interactions such as pursuit of one animal by another (in prey capture, mating, aggressive territorial battles, …)

Purpose may be – colony formation e.g. honeybees, foraging for food e.g. honeybees seeking nectar sources, avoiding predation e.g. wildebeest avoiding lions, cooperative herding of prey e.g. dolphins herding

Collective strategies of note – polarized flock movement, milling, boundary following, helical spiraling

The inference of feedback laws from sampled trajectory data is an ill-posed problem. Progress is made by regularization.

Starlings fit into the category of "animals with free will". We should not ignore the insights from behavior of dragonflies in territorial battles associated with very sophisticated visual processing. They are highly efficient and voracious eaters of fruit-flies.

Outline

- Models of Individual agents (self-steering particles, particles in matrix manifolds)
- Dyadic Interactions (pursuit, escape, boundary following)
- Models of Collectives (graphs, collective strategies, dynamics)
- Configuration Space Methods (shape space, ensemble moment of inertia, energy splitting)
- Data Assimilation (optimal data fitting, cross-validation, extraction of rules for natural collectives).

Models of individual agents are constructed as self-steering particles in 2D (or 3D) with curvatures as controls. A natural frame representation of a curve is interpreted as a control system representation on a matrix Lie group SE(2) (or SE(3)).

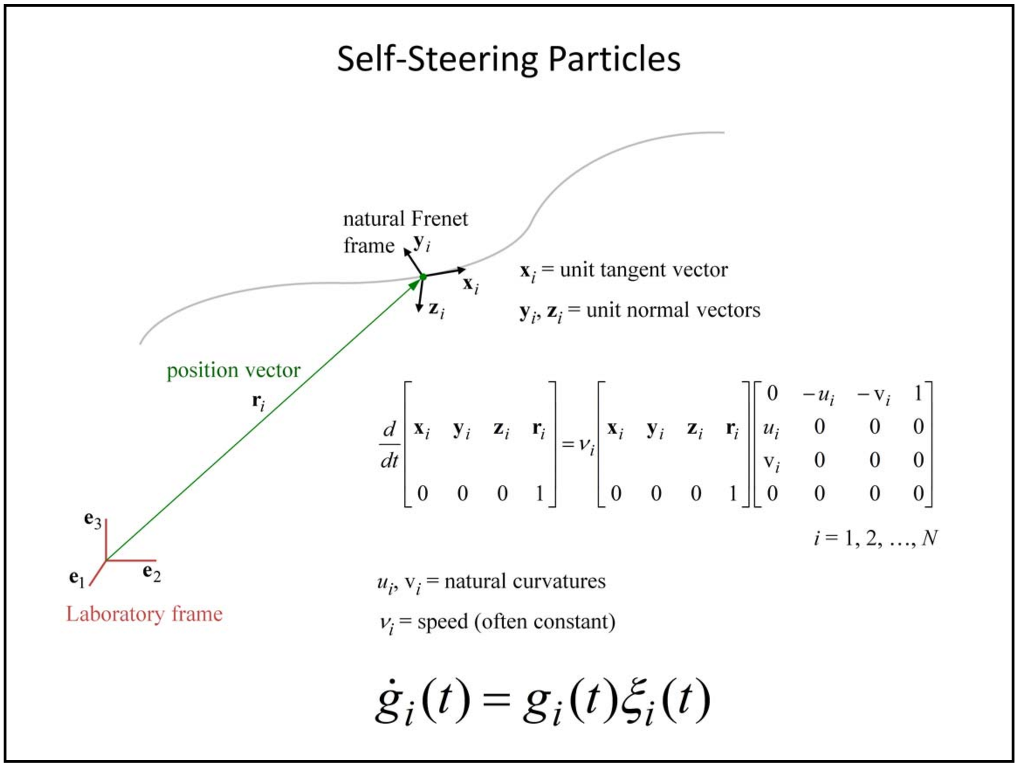
Classic dyadic interactions of agents are associated with biological activities such as pursuit, avoidance, boundary following and landmark following. These specify constraints on joint state spaces of one or more particles. A strategy is a specification of a constraint. Contrast functions measure departure from a strategy. Sensorimotor feedback laws execute strategies.

A collective of self-steering particles is specified by a graph of interactions between agents (who is attending to whom), dyadic strategies in operation, feedback laws that execute the strategies and the dynamics of attention. Taken together these elements capture the dynamics of a collective.

A top-down view of a such a collective may be derived from mechanics principles – start with configuration space, describe characteristics of configuration space and Riemannian metrics (kinetic energy quadratic forms). Analysis of data from the perspective of certain natural decompositions of velocity spaces can be revealing.

Extracting rules from sampled observations of collectives needs principled approaches to ill-posed problems. Regularization and cross-validation fit into optimal control theoretic algorithms for data assimilation.

Models of Individual Agents



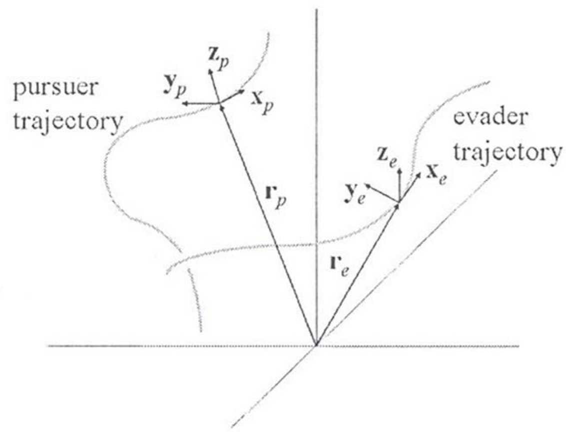
Basic model in 3 dimensions of a self-steering particle. It defines a left-invariant system. Geometric view – definition of Euclidean invariants (curvatures); Control theoretic view – definition of control inputs (curvatures).

Modeling Interactions in 3D

The **natural curvatures** are controls. The speeds are time functions dictated by propulsive/lift mechanisms.

$$\dot{\mathbf{r}}_e = \nu_e \mathbf{x}_e$$
$$\dot{\mathbf{x}}_e = \nu_e (\mathbf{y}_e u_e + \mathbf{z}_e \upsilon_e)$$
$$\dot{\mathbf{y}}_e = -\nu_e \mathbf{x}_e u_e$$
$$\dot{\mathbf{z}}_e = -\nu_e \mathbf{x}_e \upsilon_e$$

$$\dot{\mathbf{r}}_p = \nu_p \mathbf{x}_p$$
$$\dot{\mathbf{x}}_p = \nu_p (\mathbf{y}_p u_p + \mathbf{z}_p \upsilon_p)$$
$$\dot{\mathbf{y}}_p = -\nu_p \mathbf{x}_p u_p$$
$$\dot{\mathbf{z}}_p = -\nu_p \mathbf{x}_p \upsilon_p$$



$$r = r_p - r_e$$ baseline vector

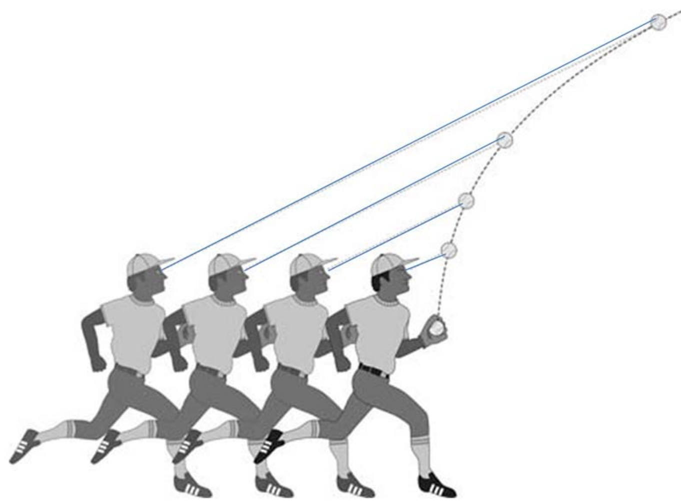
The flight behavior of a bat or a bird, or an insect, is the end result of interaction between (visual, auditory, olfactory, somatosensory, and inertial) sensing, and actuation of a complex network of muscles, mediated by the rapid and learned responses of the neural control substrate. The overwhelming richness of detail present in this feedback loop and in the physics of a multiple-degrees-of-freedom animal needs to be **abstracted** to the right level in seeking answers to questions such as: What individual behaviors govern collective cohesion? What is the structure of interaction between individuals within a collective? What organizations within a flock enable effective transmission of information across a flock? Based on the results of our prior work on 3D trajectory modeling and analysis of motion camouflage and echolocating bats (Justh and Krishnaprasad 2005; Reddy et. al. 2006, 2007; Reddy 2007; Wei et. al. 2009), we argue that a description with the right level of complexity for modeling an individual in a flock or a swarm is the Newtonian particle model.

The figure presents two particle trajectories as curves with frames, one for the evader/target (denoted as e) and one for the pursuer (denoted as p). The curvatures u and v are controls. The speeds denoted by Greek letter nu are decided by propulsive/lift considerations.

This representation of individual trajectory dynamics is known as the natural frame representation, made better known through a well-known paper of R. L. Bishop (1975). Instead of writing Newton’s equations as “ma = f” , we are making explicit the role of curvature/steering controls as inputs.

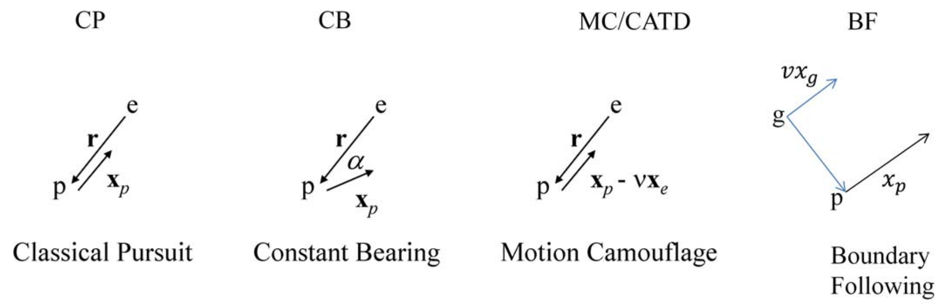
Dyadic Interactions

Gaze Heuristic



Gerd Gigerenzer (2007)

Strategies as constraint manifolds



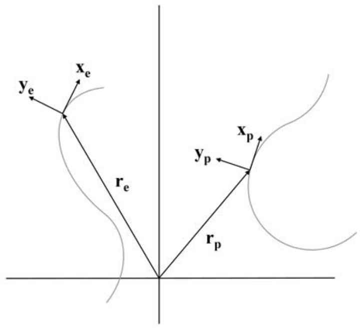
Contrast functions

$$\Lambda_0 = \mathbf{x}_p \cdot \frac{\mathbf{r}}{|\mathbf{r}|} \qquad \Lambda = R(\alpha)\mathbf{x}_p \cdot \frac{\mathbf{r}}{|\mathbf{r}|} \qquad \Gamma = \frac{\mathbf{r}}{|\mathbf{r}|} \cdot \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$$

$$R(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

We define a control strategy as the **specification** of a constraint manifold in the joint state space of the pursuer (p) and the target (e). We suggest some typical pursuit strategies.
Classical pursuit is the constraint of heading straight for the target.
Constant bearing pursuit is heading for the target with a fixed lead or lag (angle alpha).
In **3D** we need a **cone condition**.
Motion camouflage (with respect to infinity) is a stealthy pursuit, nulling motion parallax, suggested by the trajectories of dragonflies.
Motion camouflage with respect to infinity is the same as a strategy adopted by bats in pursuit of insects. In that context, we refer to it as the constant absolute target direction strategy (**CATD**).
A pursuer executes a feedback law that (approximately) fulfills the specification -
Pursuer reaches an **epsilon neighborhood** of a constraint manifold in finite time;
Pursuer converges to constraint manifold asymptotically.

Modeling Planar Motion Camouflage (1)



$$
\begin{aligned}
\dot{\boldsymbol{r}}_p &= \boldsymbol{x}_p \\
\dot{\boldsymbol{x}}_p &= \boldsymbol{y}_p u_p \\
\dot{\boldsymbol{y}}_p &= -\boldsymbol{x}_p u_p
\end{aligned}
\qquad (1)
$$

$$
\begin{aligned}
\dot{\boldsymbol{r}}_e &= \nu \boldsymbol{x}_e \\
\dot{\boldsymbol{x}}_e &= \nu \boldsymbol{y}_e u_e \\
\dot{\boldsymbol{y}}_e &= -\nu \boldsymbol{x}_e u_e
\end{aligned}
\qquad (2)
$$

Here we specialize the models of interaction to the plane. The speed ratio is given by ν, assumed constant and less than 1 in what follows.

The MC model – pursuit model; speed ratio is given by nu.

Modeling Motion Camouflage (2)

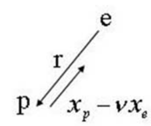
Motion camouflage with respect to a point at infinity is given by

$$r \triangleq r_p - r_e = \lambda r_\infty \quad (3)$$

Where r_∞ is a fixed unit vector and λ is a time-dependent scalar.
Infinitesimally, motion camouflage with respect to ∞ is equivalent to

$$\begin{aligned} w &\equiv \dot{r} - \left(\frac{r}{|r|}\cdot\dot{r}\right)\frac{r}{|r|} \quad (4) \\ &\equiv 0 \end{aligned}$$

(here we restrict to non-collision states, i.e. $|r| \neq 0$)



w denotes the transverse relative velocity.

Contrast Function

Let $\quad \Gamma \triangleq \left(\frac{d}{dt}|r|\right) \Big/ \left|\frac{dr}{dt}\right| \qquad (6)$

$$= \frac{r}{|r|} \cdot \frac{\dot{r}}{|\dot{r}|}$$

well-defined on non-collision states.

Observe $\quad -1 \le \Gamma \le 1 \quad , \quad 1-\nu \le |\dot{r}| \le 1+\nu$

and $\quad 1-\Gamma^2 = \frac{|w|^2}{|\dot{r}|^2}$

Driving Γ to ± 1 corresponds to reducing distance to motion camouflage manifold

As $\quad \Gamma \to +1 \quad$ baseline lengthening
As $\quad \Gamma \to -1 \quad$ baseline shortening

Finding a Feedback Law (1)

Compute $\dot{\Gamma}$ (as a function of $u_p,\ u_e$ etc.)

Find law for u_p to make $\dot{\Gamma}$ negative (in a suitable region).

Consider

$$\boxed{u_p = -\mu\left(\frac{r}{|r|}\cdot\dot{r}^{\perp}\right) + \left[\frac{\left(x_p\cdot x_e\right)-\nu}{1-\nu\left(x_p\cdot x_e\right)}\right]\nu^2 u_e} \qquad (7)$$

For any $\mu > 0$ there exists $r_0 > 0$ such that under (7),

$$\dot{\Gamma} \le 0, \qquad \forall\, |r| > r_0$$

But, second term in (7) asks for too much information.

Finding a Feedback Law (2)

Consider keeping first term only in (7), but add hypothesis that $|u_e|$ is bounded. Thus

$$\boxed{u_p = -\mu\left(\frac{r}{|r|}.\dot{r}^{\perp}\right)} \qquad (8)$$

Definition

For the pursuit-evader system (1), (2) with Γ defined by (6), we say that motion camouflage is <u>accessible in finite time</u> if for any $\varepsilon > 0$, there exists a time $t_1 > 0$ such that

$$\Gamma(t_1) \le -1 + \varepsilon$$

E.W. Justh and P. S. Krishnaprasad (2006), *Proc. R. Soc. A*, 462:3629-3643.
P.V. Reddy, E.W. Justh and P. S. Krishnaprasad (2006), *45th IEEE CDC*, pp.3327-3332.

High Gain Feedback

Proposition 1: For system (1)(2), Γ as in (6) and control law (8), with the following hypotheses:

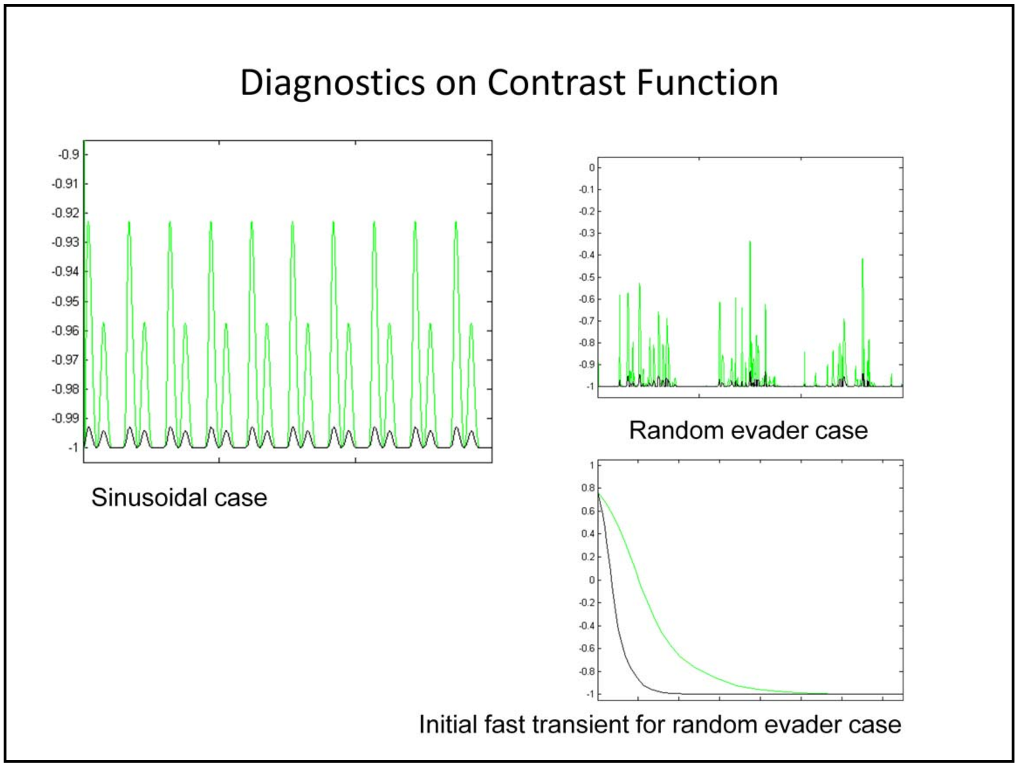
(A1) $0 < v < 1$ (and v is constant)

(A2) u_e is continuous and $|u_e|$ is bounded

(A3) $|r(0)| > 0$ and

(A4) $\Gamma_0 = \Gamma(0) < 1$

Then motion camouflage is accessible in finite time using high-gain feedback (i.e., choosing $\mu > 0$ sufficiently large).



The green curves correspond to lower gain values; the blue curves correspond to higher gain values. In the panel on lower right the time scale is stretched to see the initial transient of Gamma decreasing down to nearly -1.

Stochasticity and Accessibility

Proposition 2 (Galloway-Justh-K, 2007):

Consider the system (1) - (2), with control law (8), and Γ defined by (6), with the following hypotheses:
(A1) 0 < ν < 1 (and is constant),

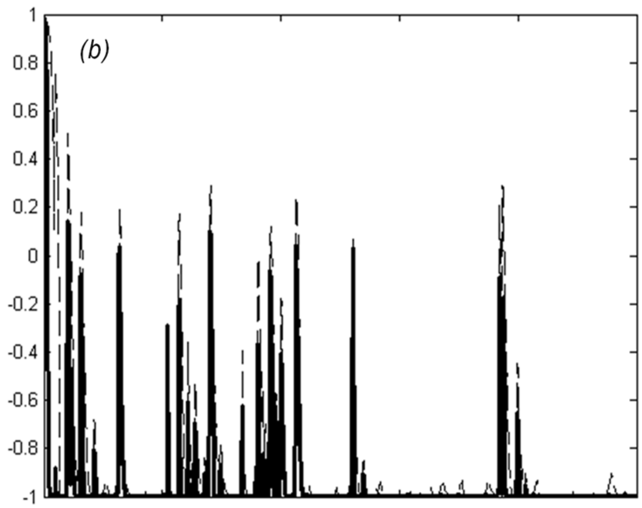
(A2) u_e is a stochastic process with piecewise continuous sample paths and bounded first and second moments

(A3) u_e is of a form such that the matrix X = $[x_e \quad y_e]$ evolves on SO(2),

(A4) Initial conditions are generic;

Then motion camouflage is **accessible in the mean** in finite time using high-gain feedback (i.e., by choosing μ > 0 sufficiently large.)

Contrast Function Excursions



3 state run-and-tumble evader – mean straight durations 40 times longer than turn durations

Biological Data

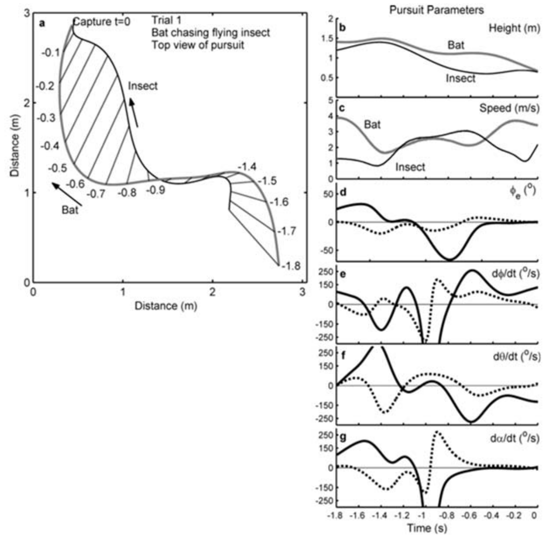
Data from the Batlab (courtesy of Prof. Cynthia Moss); strategies of insect prey capture (CATD - constant absolute target direction strategy same as MC)

Biological Data

Movie of echolocating bat *E. fuscus* tracking and capturing free-flying praying mantis *P. agrionina* (courtesy of Prof. Cynthia F. Moss, University of Maryland) – CATD/MC strategy at work

Prey Capture Movie (Ghose & Moss)

Figure 2. Bat Chasing a Flying Insect



Ghose K, Horiuchi TK, Krishnaprasad PS, Moss CF (2006) Echolocating Bats Use a Nearly Time-Optimal Strategy to Intercept Prey. PLoS Biol 4(5): e108. doi:10.1371/journal.pbio.0040108
http://www.plosbiology.org/article/info:doi/10.1371/journal.pbio.0040108

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Summarizing the Data

From mariners avoiding collision courses, to baseball outfielders catching flyballs, the **constant bearing** (CB) strategy has been known as an effective strategy.

Here we see bats executing a **different** strategy – keeping **constant absolute target direction** (CATD), geometrically indistinguishable from what we referred to earlier as **motion camouflage with respect to infinity.**

K. Ghose and C.F. Moss (2006), *J. Neuroscience*, 26(6):1704-1710.
K. Ghose, T.K. Horiuchi, P.S. Krishnaprasad and C.F. Moss (2006), *PLoS Biology*, 4(5), 865-873, e108.

BUT context influences strategy – see next.
Confirmed by a re-analysis by Biswadip Dey using refined data assimilation techniques

Biological Data – Competing Bats

3348

The Journal of Experimental Biology 213, 3348-3356

doi:10.1242/jeb.044818

Effects of competitive prey capture on flight behavior and sonar beam pattern in paired big brown bats, *Eptesicus fuscus*

Chen Chiu[1,2,*], Puduru Viswanadha Reddy[3], Wei Xian[1], Perinkulam S. Krishnaprasad[2,4] and Cynthia F. Moss[1,2]

[1]Department of Psychology, University of Maryland, College Park, MD 20742, USA, [2]Institute for Systems Research, University of Maryland, College Park, MD 20742, USA, [3]Department of Econometrics and Operations Research, K519, Tilburg University, PO Box 90153, 5000 LE Tilburg, Netherlands and [4]Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742, USA

*Author for correspondence (chiuchen@gmail.com)

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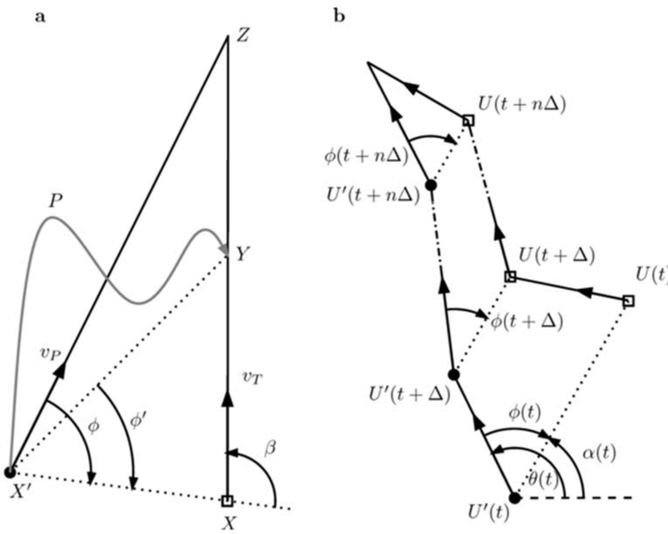
SUMMARY

Foraging and flight behavior of echolocating bats were quantitatively analyzed in this study. Paired big brown bats, *Eptesicus fuscus*, competed for a single food item in a large laboratory flight room. Their sonar beam patterns and flight paths were recorded by a microphone array and two high-speed cameras, respectively. Bats often remained in nearly classical pursuit (CP) states when one bat is following another bat. A follower can detect and anticipate the movement of the leader, while the leader has the advantage of gaining access to the prey first. Bats in the trailing position throughout the trial were more successful in accessing the prey. In this study, bats also used their sonar beam to monitor the conspecific's movement and to track the prey. Each bat tended to use its sonar beam to track the prey when it was closer to the worm than to another bat. The trailing bat often directed its sonar beam toward the leading bat in following flight. When two bats flew towards each other, they tended to direct their sonar beam axes away from each other, presumably to avoid signal jamming. This study provides a new perspective on how echolocating bats use their biosonar system to coordinate their flight with conspecifics in a group and how they compete for the same food source with conspecifics.

Figure 1. Time-Optimal Strategies to Intercept a Target



Ghose K, Horiuchi TK, Krishnaprasad PS, Moss CF (2006) Echolocating Bats Use a Nearly Time-Optimal Strategy to Intercept Prey. PLoS Biol 4(5): e108. doi:10.1371/journal.pbio.0040108
http://www.plosbiology.org/article/info:doi/10.1371/journal.pbio.0040108

$$\phi = sin^{-1}\left(\frac{v_T \sin\beta}{v_P}\right) \qquad (1)$$

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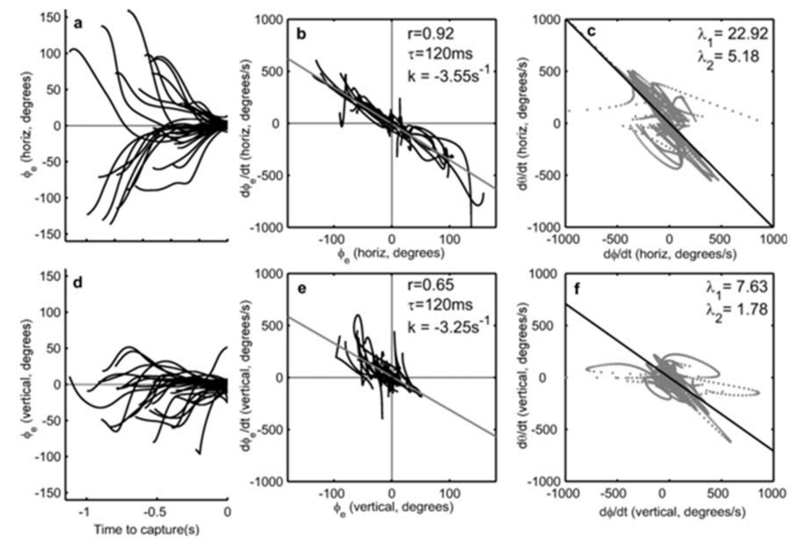
Figure 5.



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http://www.plosbiology.org/article/info:doi/10.1371/journal.pbio.0040108

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We define for every instant t,

$$\phi_e(t) = \phi(t) - \phi_{opt}(t) \quad (2)$$

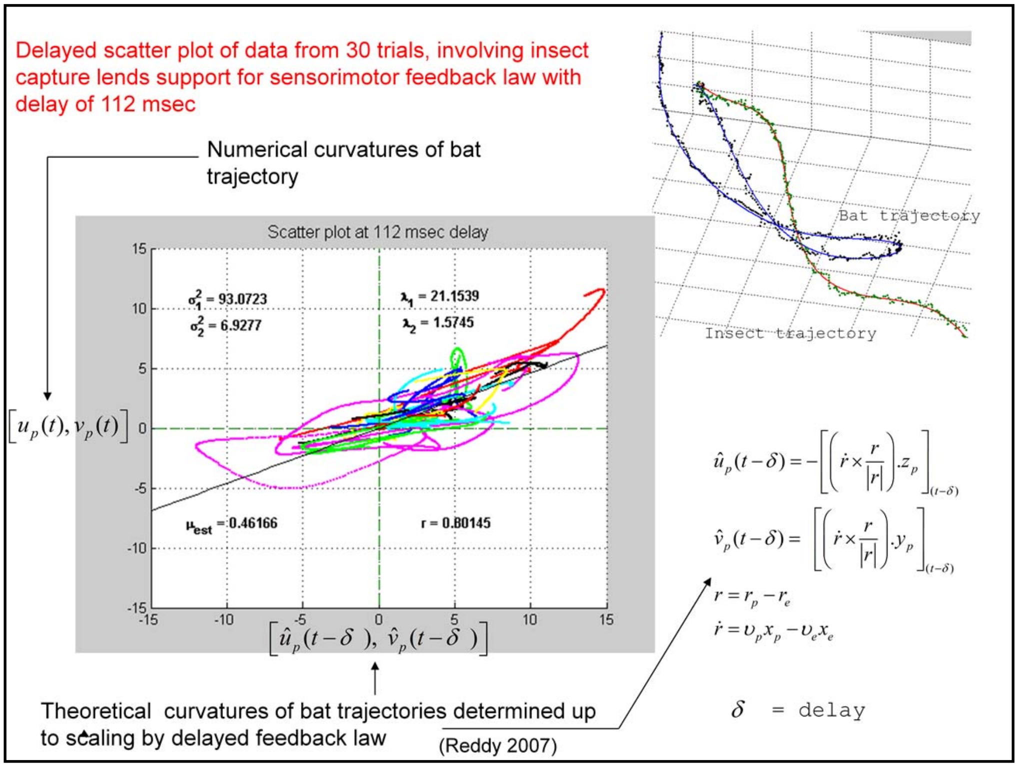
the difference between the actual bearing to the target, $\phi(t)$, and the optimum bearing, $\phi_{opt}(t)$, given by Equation 1.

$$\phi = sin^{-1}\left(\frac{v_T \sin\beta}{v_P}\right) \quad (1)$$

To determine if the bat's flight behavior was better described by the CB strategy or the CATD strategy, we analyzed 30 successful insect captures by eight bats. Of these, 15 trials were of the bat capturing free-flying insects, and 15 trials were of the bat capturing tethered insects (Figure 5). In each case the bat was observed to maneuver to approach the optimum bearing in both horizontal and vertical planes (Figure 5A and 5D). As can be seen from the plots of $d\phi_e/dt$ against ϕ_e in Figure 5B and 5E, the bat maneuvered to reduce ϕ_e to zero during pursuit. We were able to model the ϕ_e data well by a delay-differential equation

$$\frac{d\phi_e(t)}{dt} = k\phi_e(t - \tau) \quad (5)$$

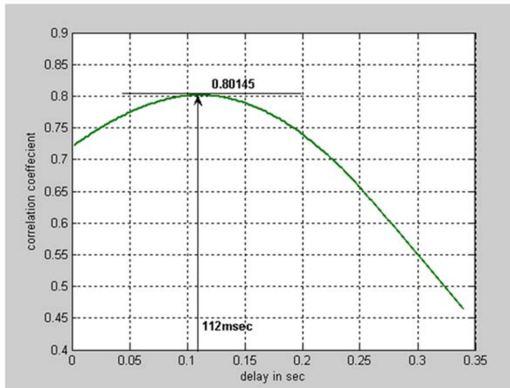
with a negative gain parameter k and a delay τ. The delay, τ, in the model is most likely due to a combination of delays in different parts of the system, including sensorimotor processing time and delay due to the aerodynamics of the bat. It follows from the theory of delay differential equations [18] that solutions to Equation 5 are well-posed and unique given any initial condition, $\phi_e^{initial}(t)$, over a time interval of length τ. Moreover, if the gain k is negative and the product $k\tau$ of the gain and time delay is greater than $-\pi/2$, each solution is a weighted infinite sum of decaying exponentials, and the decay rate of each term in the sum is given by the roots of the characteristic exponential polynomial $s - ke^{-\tau s}$ associated with the delay differential equation (Equation 5) (see Theorems 4.1 and 13.8 in [18], a result due to Hayes [19]. This stability constraint on the parameters of the model is met by the estimates of k and τ in Figures 5B and 5E.



This figure summarizes the outcome of a statistical examination of the hypothesis that in insect pursuit, a bat uses a delayed feedback law that is linear in the rate of rotation of the baseline from the bat to the insect. Video data from two infra-red cameras in the flight room was used to obtain trajectory data for insect and bat at a sampling rate of once every 4 msec. Using an optimization method for regularization of ill-conditioned problems, we obtained numerically the instant-by-instant trajectory curvatures of the bat, from the sampled trajectories. These numerical curvatures are plotted above against a delayed version of the hypothetical feedback law, where the delay accounts for the overall latency in the response of the bat to changes in the flight of the insect, including sensorimotor neural computation, biomechanical delay and aerodynamics. A range of delays was considered, and the best delay (in the sense of maximum correlation, here 0.80145), turned out to be about 112 msec. This best delay is consistent with other known estimates in the literature. The results show the effectiveness of the pursuit strategy employed by the bat, based on directional and target range cues obtained by biosonar even in the presence of delay. The technological implications of similar sensorimotor strategies in robotic assist systems for humans are being explored through experiments in the Computational Sensorimotor Systems Laboratory, and the Intelligent Servosystems Laboratory, of the University of Maryland.

Statistical Evaluation

$$\rho_{[u_p(t),v_p(t)][\hat{u}_p(t-\delta),\hat{v}_p(t-\delta)]}$$



P.V. Reddy, M.S. Thesis 2007

Delay vs. correlation

Accessibility with Delay

<u>Theorem</u>: Consider the planar pursuit system with speed ratio ν high gain control law with gain μ, and feedback delay τ.
Suppose the following hypotheses hold:

- (A1) $0 < \nu < 1$
- (A2) u_e is continuous and $|u_e| < u_{max}$.
- (A3) $\Gamma_0 = \Gamma(0) < 1 - \varepsilon$
- (A4) $|r(0)| > 0$

Let $0 < r_0 < |r(0)|$ and $T = \frac{|r(0)| - r_0}{1+\nu}$,
$\delta = max\left(0, \frac{1}{2}\left(\frac{1+\nu}{|r(0)|-r_0}\right) ln\left[\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right)\left(\frac{2-\varepsilon}{\varepsilon}\right)\right]\right)$.

Let $c_0 = \mu\left(\frac{1-\nu}{1+\nu}\right) - \frac{1+\nu}{r_0}$; $\quad c_1 = \frac{\nu^2(1+\nu)u_{max}}{(1-\nu)^2}$

Suppose $\mu > \frac{(1+\nu)^2}{(1-\nu)r_0}$ and $c_2 = c_0 - \frac{1}{\sqrt{\varepsilon}}\left[c_1 + \tau\mu\left(c_1 + \left(\frac{1+\nu}{1-\nu}\right)^2\left(\mu + \frac{1+\nu}{2r_0}\right)\right)\right] > \delta$

Then there exists a finite time t_1, for which $\Gamma(t_1) \leq -1 + \varepsilon$.

Reddy-Justh-K, 2007

This theorem gives conditions for the existence of a nonempty region of delay-gain pairs which allow finite time accessibility of the motion camouflage manifold. Reddy et. al. (2007).

Gain-Delay Tradeoff via Peak Deviation

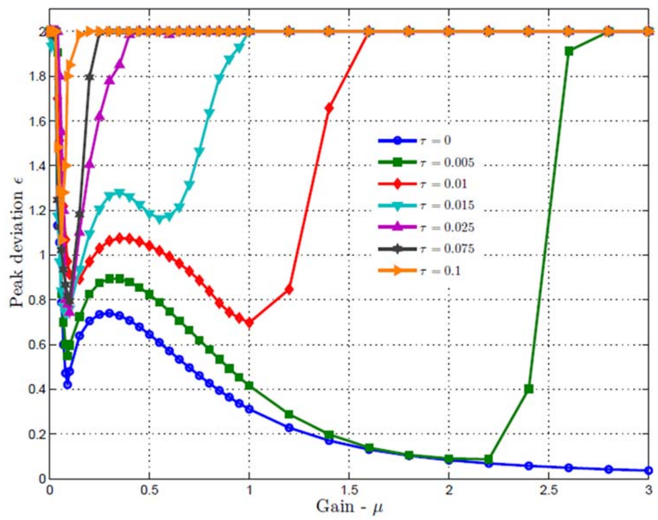


Figure 4.7: Peak deviation from motion camouflage (ϵ) is plotted as a function of increasing feedback gain in the presence of various delays.

Gain-Delay Tradeoff via Peak Deviation

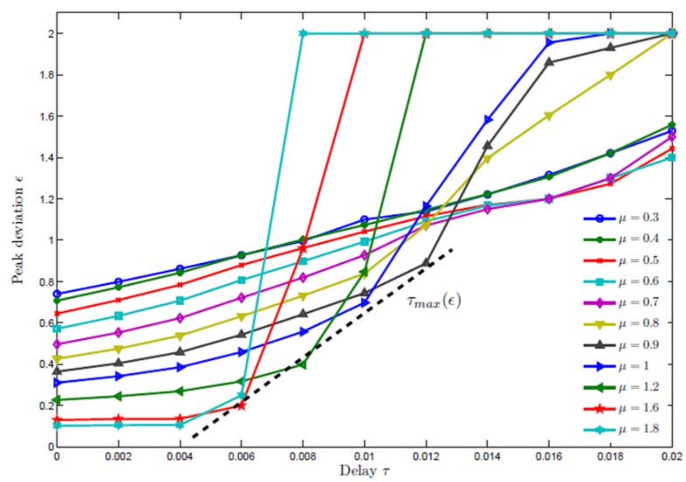


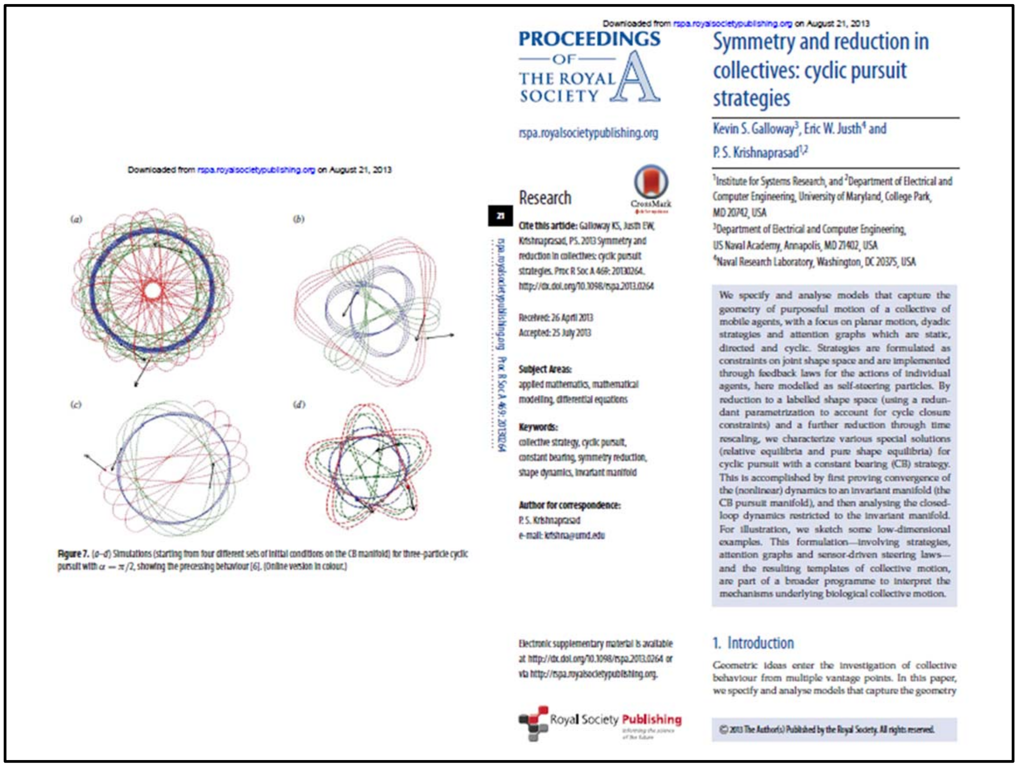
Figure 4.8: Peak deviation from motion camouflage (ϵ) is plotted as a function of increasing delay for various feedback gains. The dark dashed line corresponds to $\tau_{max}(\epsilon)$, an upper bound on the delay that can be introduced into the system for a given ϵ.

Remarks

- Gain versus Delay tradeoffs can be carried to the finite time behavior settings
- The resulting tradeoff curves are markedly different from what we expect in linear setting of asymptotic stability analysis
- Related tradeoffs incorporating stochasticity would be of interest

Models of Collectives

Movie of flocking event of European starling *Sturnus vulgaris* over
Rome (courtesy of Dr. Andrea Cavagna)



(a) (b) (c) (d)

Figure 7. (a–d) Simulations (starting from four different sets of initial conditions on the CB manifold) for three-particle cyclic pursuit with $\alpha = \pi/2$, showing the precessing behaviour [6]. (Online version in colour.)

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Research

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collective strategy, cyclic pursuit, constant bearing, symmetry reduction, shape dynamics, invariant manifold

Author for correspondence:
P. S. Krishnaprasad
e-mail: krishna@umd.edu

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Symmetry and reduction in collectives: cyclic pursuit strategies

Kevin S. Galloway[3], Eric W. Justh[4] and P. S. Krishnaprasad[1,2]

[1]Institute for Systems Research, and [2]Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742, USA
[3]Department of Electrical and Computer Engineering, US Naval Academy, Annapolis, MD 21402, USA
[4]Naval Research Laboratory, Washington, DC 20375, USA

We specify and analyse models that capture the geometry of purposeful motion of a collective of mobile agents, with a focus on planar motion, dyadic strategies and attention graphs which are static, directed and cyclic. Strategies are formulated as constraints on joint shape space and are implemented through feedback laws for the actions of individual agents, here modelled as self-steering particles. By reduction to a labelled shape space (using a redundant parametrization to account for cycle closure constraints) and a further reduction through time rescaling, we characterize various special solutions (relative equilibria and pure shape equilibria) for cyclic pursuit with a constant bearing (CB) strategy. This is accomplished by first proving convergence of the (nonlinear) dynamics to an invariant manifold (the CB pursuit manifold), and then analysing the closed-loop dynamics restricted to the invariant manifold. For illustration, we sketch some low-dimensional examples. This formulation—involving strategies, attention graphs and sensor-driven steering laws—and the resulting templates of collective motion, are part of a broader programme to interpret the mechanisms underlying biological collective motion.

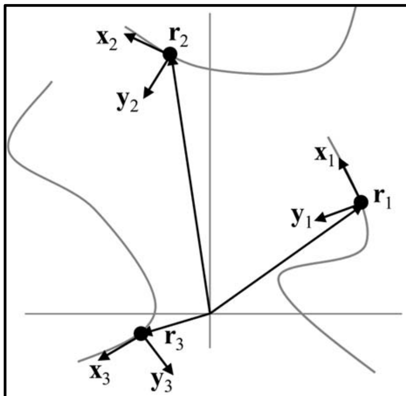
1. Introduction

Geometric ideas enter the investigation of collective behaviour from multiple vantage points. In this paper, we specify and analyse models that capture the geometry

Recent research in collective behavior demonstrates the power of geometric thinking (symmetries, shapes, model reduction and nonlinear dynamics) in elucidating collective behaviors observed in nature and realizable in the laboratory. On the left you see figures corresponding to a 3 body problem modeling interactions via constant bearing pursuit with a fixed cycle graph of attention. For chosen parameter values, periodic solutions arise in a two dimensional phase space producing quasi-periodic motions in physical space (shown). Some related work is being used to explore data on starling flocks.

Collectives – a model problem

symmetries of the n particle constant bearing (CB) cyclic pursuit problem (including time re-scaling or self-similarity); the n= 3 symmetric case and phase portraits; physical space animations of periodic orbits for the case alpha = pi/2



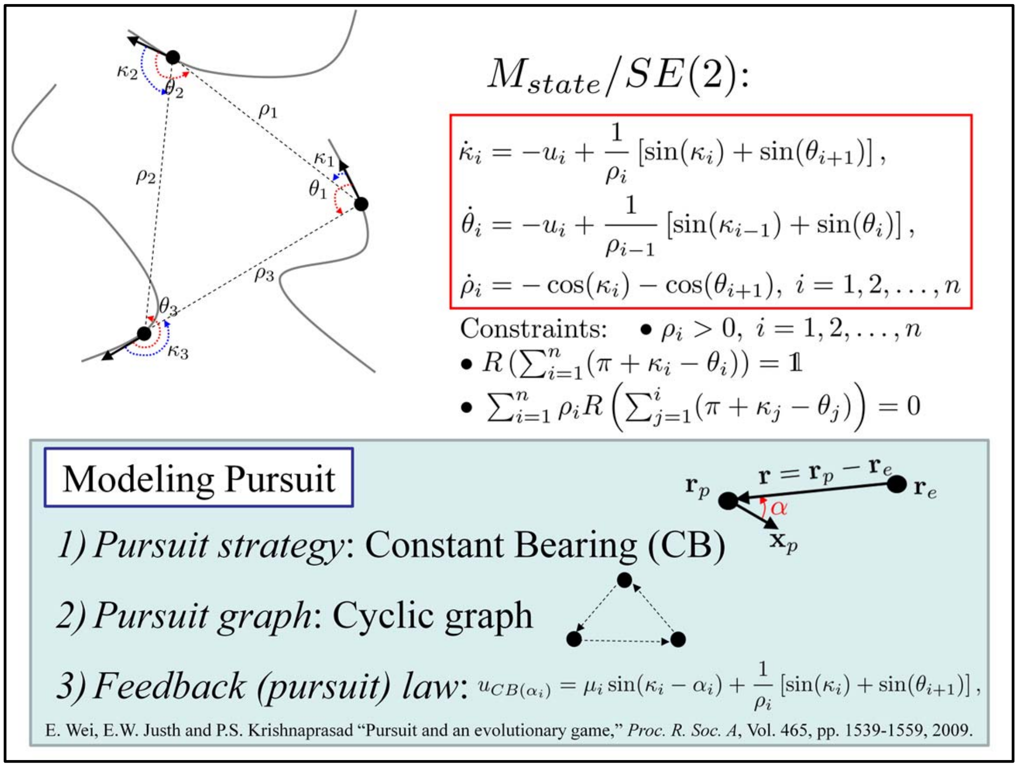
M_{state}:

$$\boxed{\begin{aligned} \dot{\mathbf{r}}_i &= \mathbf{x}_i, \\ \dot{\mathbf{x}}_i &= \mathbf{y}_i u_i, \\ \dot{\mathbf{y}}_i &= -\mathbf{x}_i u_i,\ i = 1, 2, \ldots, n \end{aligned}}$$

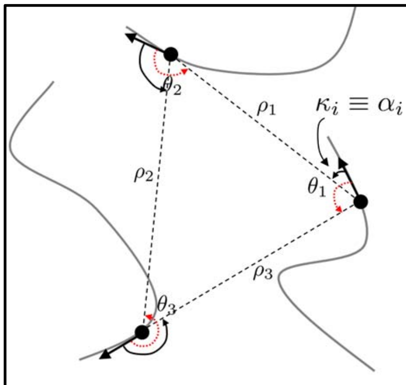
Assumptions:

1. constant (unit) speed
2. $\mathbf{r}_i \neq \mathbf{r}_{i+1}$ (i.e. "no sequential colocation")

Be clear that the dynamics do not enforce the collision prohibition



Need to make it clear up front that I'm permitting alpha to take values in [0,2pi]



CB Pursuit Manifold $M_{CB(\boldsymbol{\alpha})}$:

$$M_{CB(\boldsymbol{\alpha})} = \Big\{(\kappa_1, \theta_1, \rho_1, \ldots, \kappa_n, \theta_n, \rho_n) \in M_{state}/SE(2) \;\Big|\; \Lambda_i \triangleq -\cos(\kappa_i - \alpha_i) = -1, i = 1, 2, \ldots, n\Big\},$$

Reduced shape dynamics on $M_{CB(\boldsymbol{\alpha})}$:

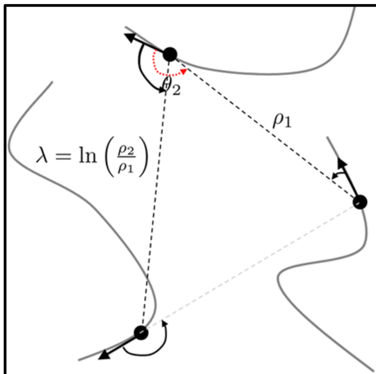
$$\dot{\theta}_i = \frac{\sin(\alpha_{i-1}) + \sin(\theta_i)}{\rho_{i-1}} - \frac{\sin(\alpha_i) + \sin(\theta_{i+1})}{\rho_i},$$

$$\dot{\rho}_i = -\cos(\alpha_i) - \cos(\theta_{i+1}),\; i = 1, 2, \ldots, n$$

Constraints:
- $\rho_i > 0,\; i = 1, 2, \ldots, n$
- $R\left(\sum_{i=1}^{n}(\pi + \alpha_i - \theta_i)\right) = \mathbb{1}$
- $\sum_{i=1}^{n} \rho_i R\left(\sum_{j=1}^{i}(\pi + \alpha_j - \theta_j)\right) = 0$

CB manifold is an attracting invariant manifold; invariant in the sense that the closed-loop vector field when restricted to the manifold is tangential to the manifold.

Lead in to next slide: we want to separate the scale dynamics from the dynamics of the "pure shape", the shape up to similarity.



$$\dot{\theta}_2 = \frac{1}{e^{\lambda}\rho_1 P}\left\{P\left[e^{\lambda}\Big(\sin(\alpha_1)+\sin(\theta_2)\Big)-\sin(\alpha_2)\right] - \sin(\theta_2-\alpha_2-\alpha_3) - e^{\lambda}\sin(\alpha_3)\right\},$$

$$\dot{\lambda} = \frac{1}{e^{\lambda}\rho_1 P}\left\{P\left[e^{\lambda}\Big(\cos(\alpha_1)+\cos(\theta_2)\Big)-\cos(\alpha_2)\right] + \cos(\theta_2-\alpha_2-\alpha_3) - e^{\lambda}\cos(\alpha_3)\right\},$$

$$\dot{\rho}_1 = -\cos(\alpha_1)-\cos(\theta_2)$$

$$P \triangleq \sqrt{e^{2\lambda}-2e^{\lambda}\cos(\theta_2-\alpha_2)+1},$$

Three-particle case $(n=3)$

Time-scaling $\tau \triangleq \int_0^t \frac{1}{e^{\lambda(\sigma)}\rho_1(\sigma)P(\sigma)}d\sigma,$

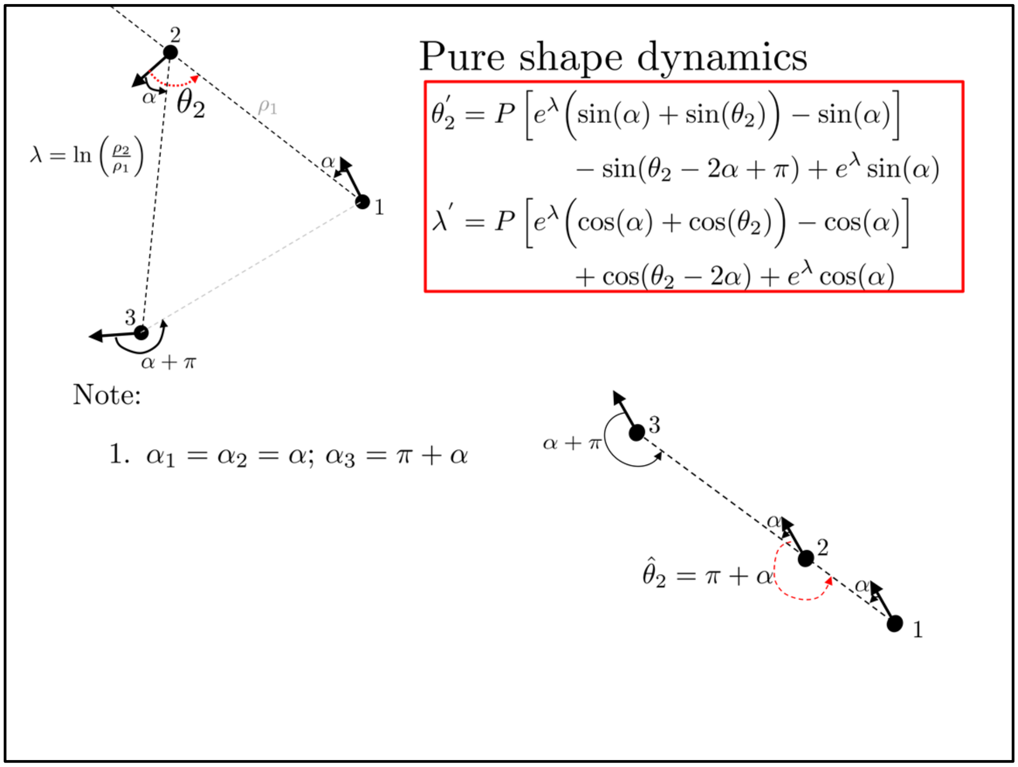
Pure shape dynamics

$$\boxed{\begin{aligned}\theta_2' &= P\left[e^{\lambda}\Big(\sin(\alpha_1)+\sin(\theta_2)\Big)-\sin(\alpha_2)\right]-\sin(\theta_2-\alpha_2-\alpha_3)-e^{\lambda}\sin(\alpha_3)\\ \lambda' &= P\left[e^{\lambda}\Big(\cos(\alpha_1)+\cos(\theta_2)\Big)-\cos(\alpha_2)\right]+\cos(\theta_2-\alpha_2-\alpha_3)-e^{\lambda}\cos(\alpha_3)\end{aligned}}$$

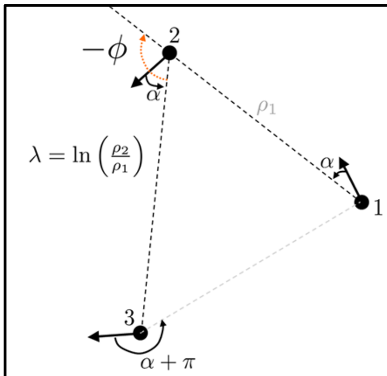
$$\rho_1' = -e^{\lambda}\rho_1 P\left[\cos(\alpha_1)+\cos(\theta_2)\right]$$

Let $\alpha_1 = \alpha_2 = \alpha = \pi + \alpha_3$, for some $\alpha \in [0, 2\pi)$

This reduction is possible for arbitrary n
Absorb the constraints
P
Time-scaling
In this case, it yields two-dimensional dynamics, which can be analyzed by phase portraits
Parametrized by the three CB parameters alpha_1, alpha_2, alpha_3
Need to mention something about other cases for alpha (maybe a brief reference to my previous CDC paper on relative equilibria)



The next slides will illustrate the alpha-dependence in a graphical way

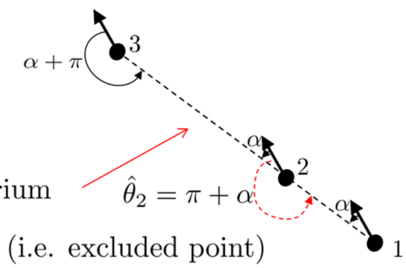


Pure shape dynamics

$$\boxed{\begin{aligned}\phi' &= P\left[e^{\lambda}\Big(\sin(\alpha) - \sin(\phi+\alpha)\Big) - \sin(\alpha)\right] \\ &\quad - \sin(\phi-\alpha) + e^{\lambda}\sin(\alpha), \\ \lambda' &= P\left[e^{\lambda}\Big(\cos(\alpha) - \cos(\phi+\alpha)\Big) - \cos(\alpha)\right] \\ &\quad + \cos(\phi-\alpha) + e^{\lambda}\cos(\alpha).\end{aligned}}$$

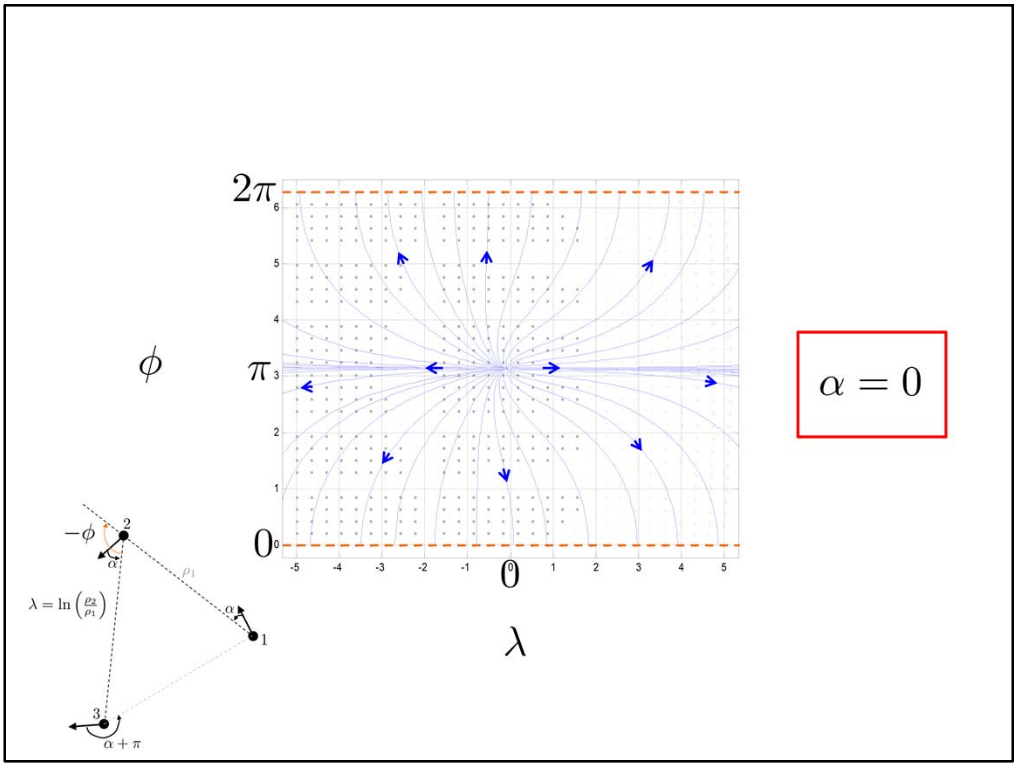
Note:

1. $\alpha_1 = \alpha_2 = \alpha$; $\alpha_3 = \pi + \alpha$
2. $\phi \triangleq \theta_2 - \hat{\theta}_2 = \theta_2 - \pi - \alpha$
3. $\phi = 0 \iff$ rectilinear equilibrium
4. $(\phi, \lambda) = (\pi, 0) \implies$ colocation (i.e. excluded point)



A family of two-dimensional dynamics (phase portraits) parametrized by $\alpha \in [0, 2\pi)$

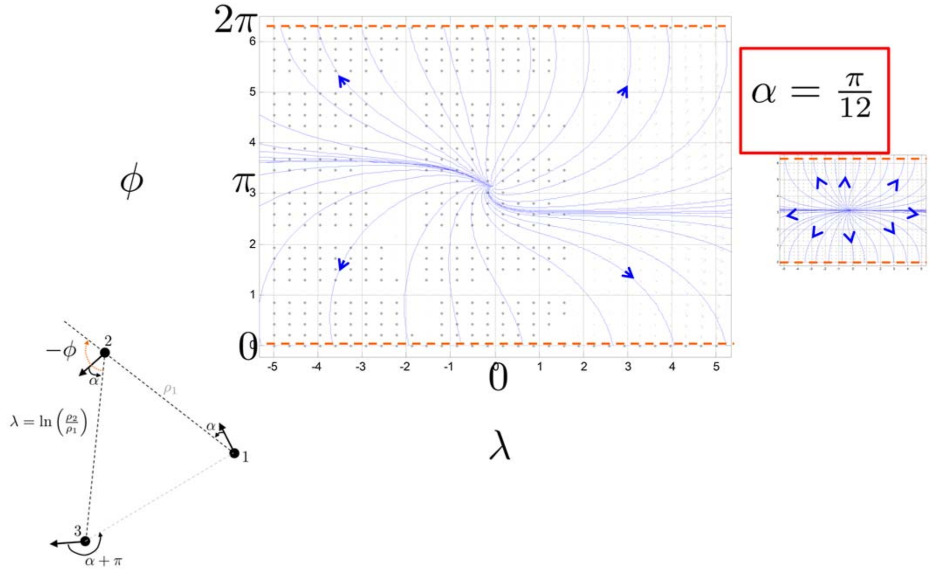
The next slides will illustrate the alpha-dependence in a graphical way

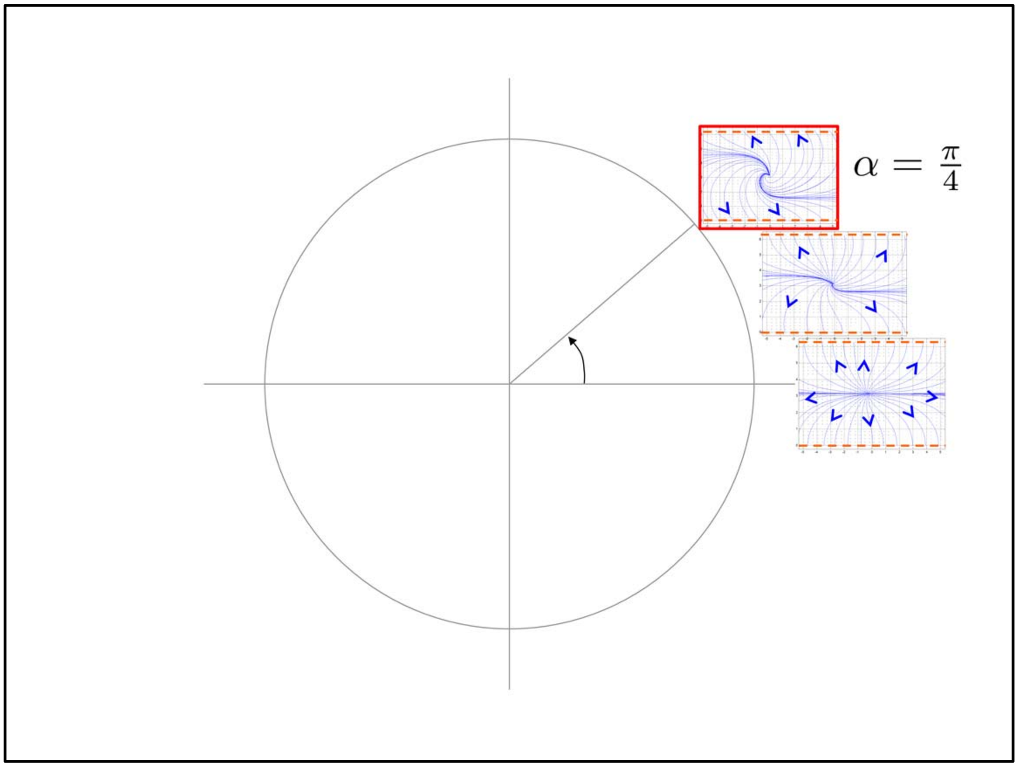


Need to point out:

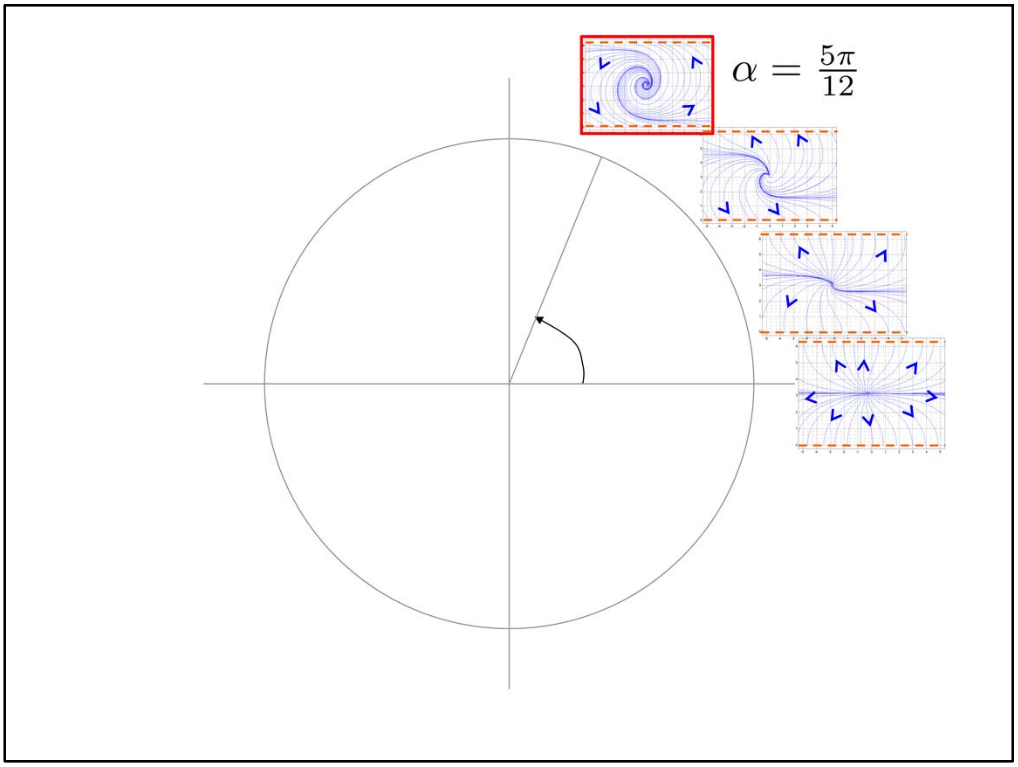
- This is an unwrapped cylinder
- Continuum of rectilinear equilibria
- It is punctured

I'll depict a series of phase portraits corresponding to values of alpha around the unit circle, starting with alpha=0
Briefly state the proposition concerning convergence to rectilinear equilibria

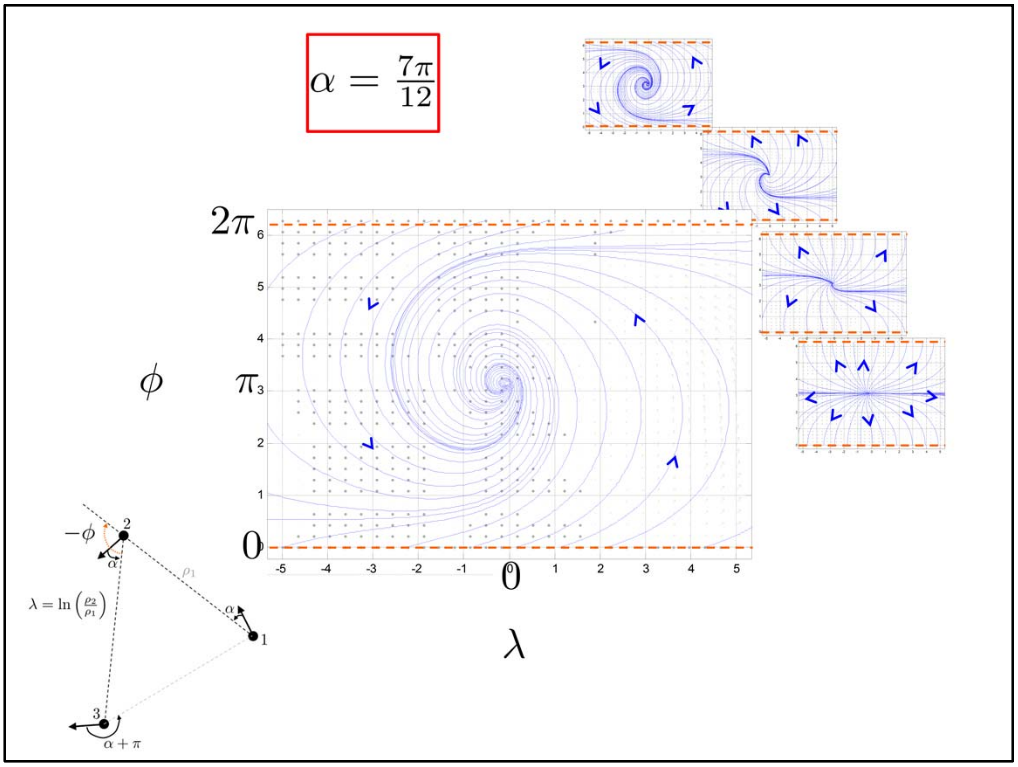

$\alpha = \frac{\pi}{12}$
ϕ
2π
π
0
0
λ
$-\phi$
$\lambda = \ln\left(\frac{\rho_2}{\rho_1}\right)$
$\alpha + \pi$



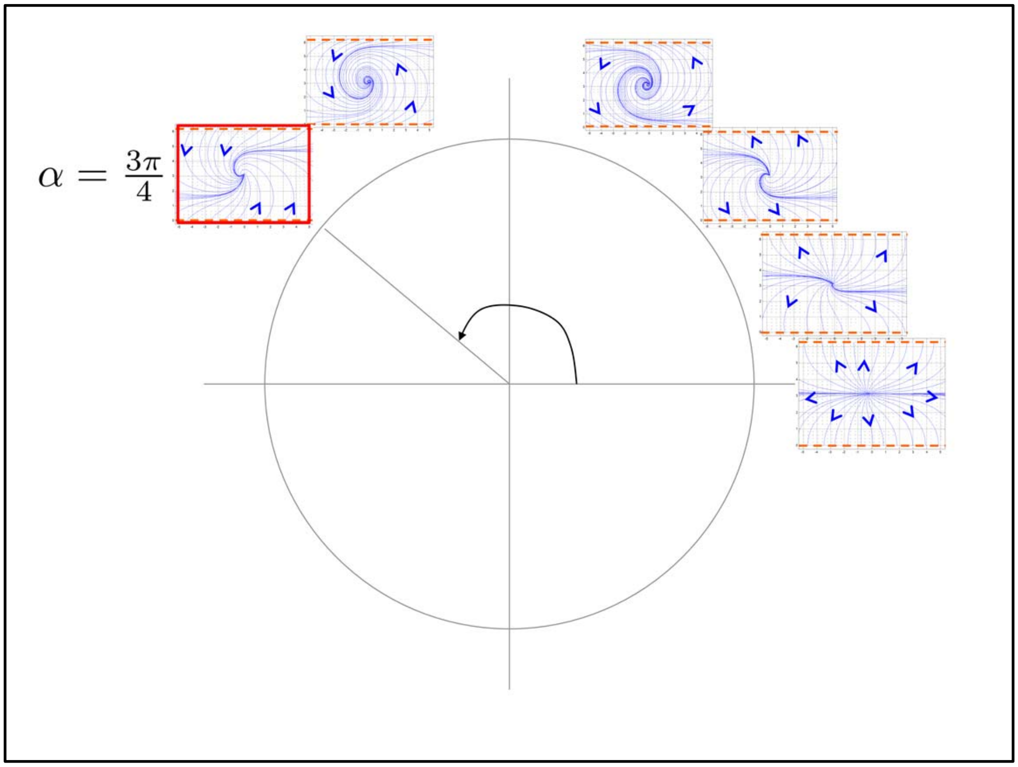
$\alpha = \frac{\pi}{4}$



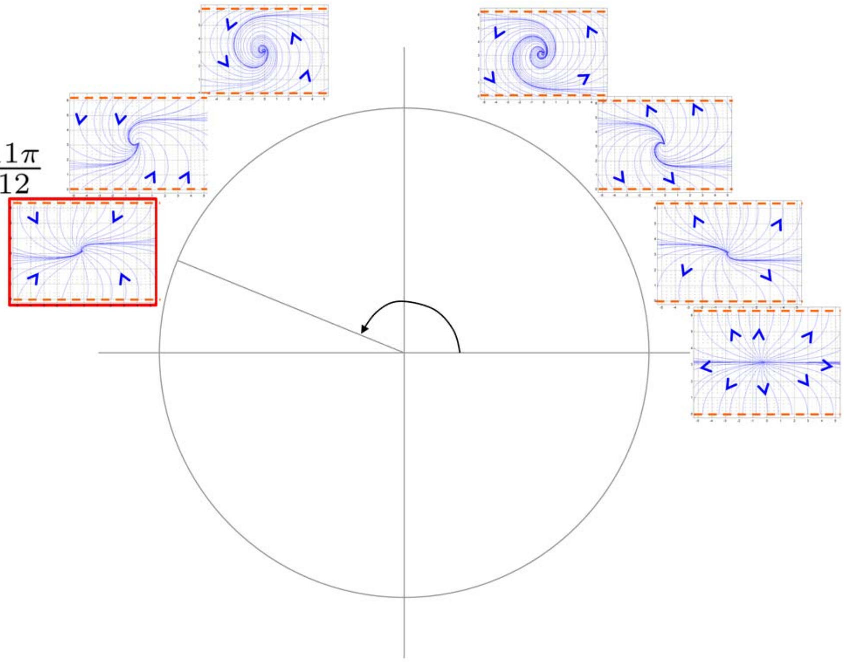
Briefly state the proposition concerning convergence to rectilinear equilibria

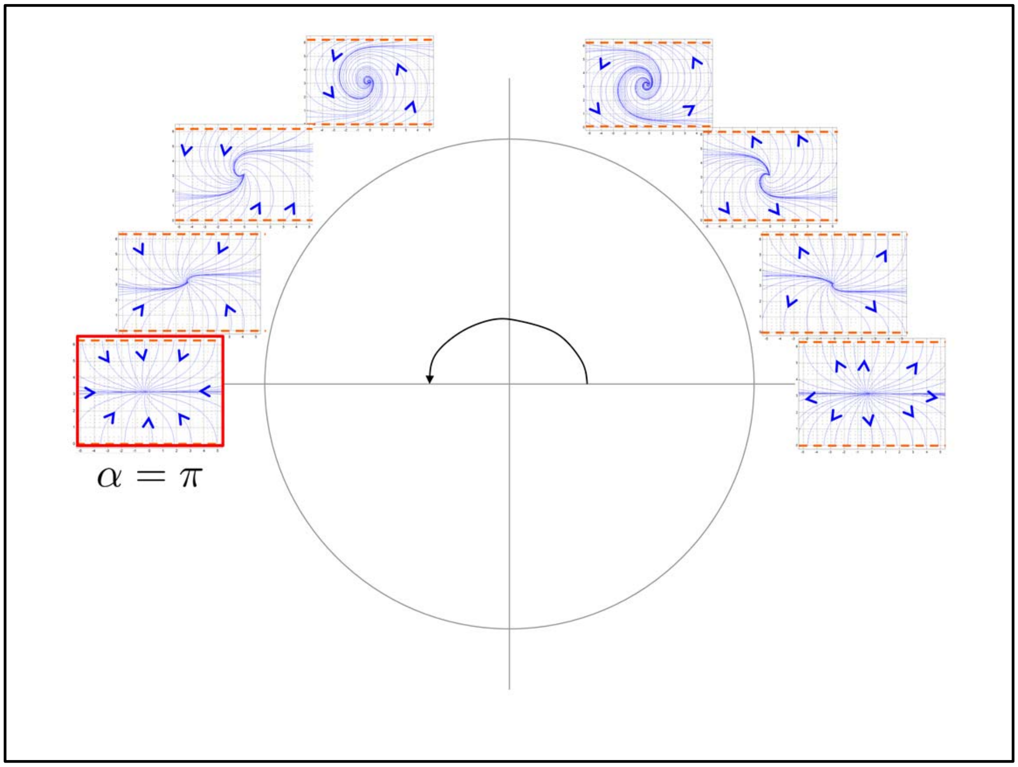


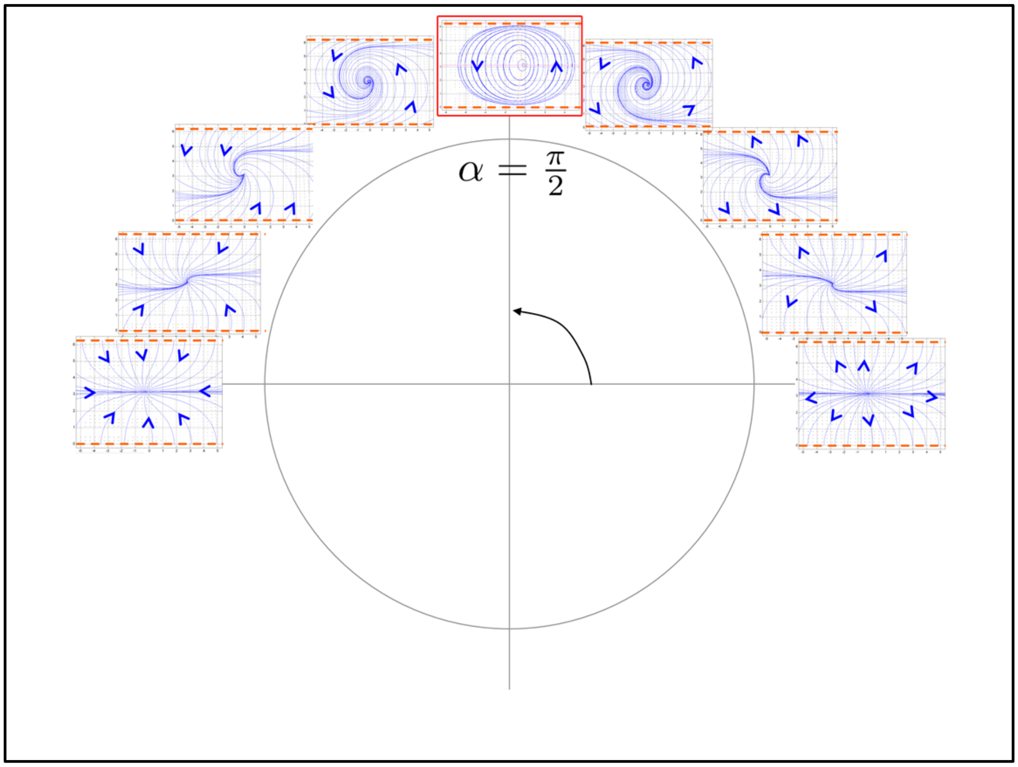
I’ll skip pi/2 for the moment, and consider alpha just greater than pi/2
These are unstable
Trajectories spiral in to the excluded point (i.e. collision)



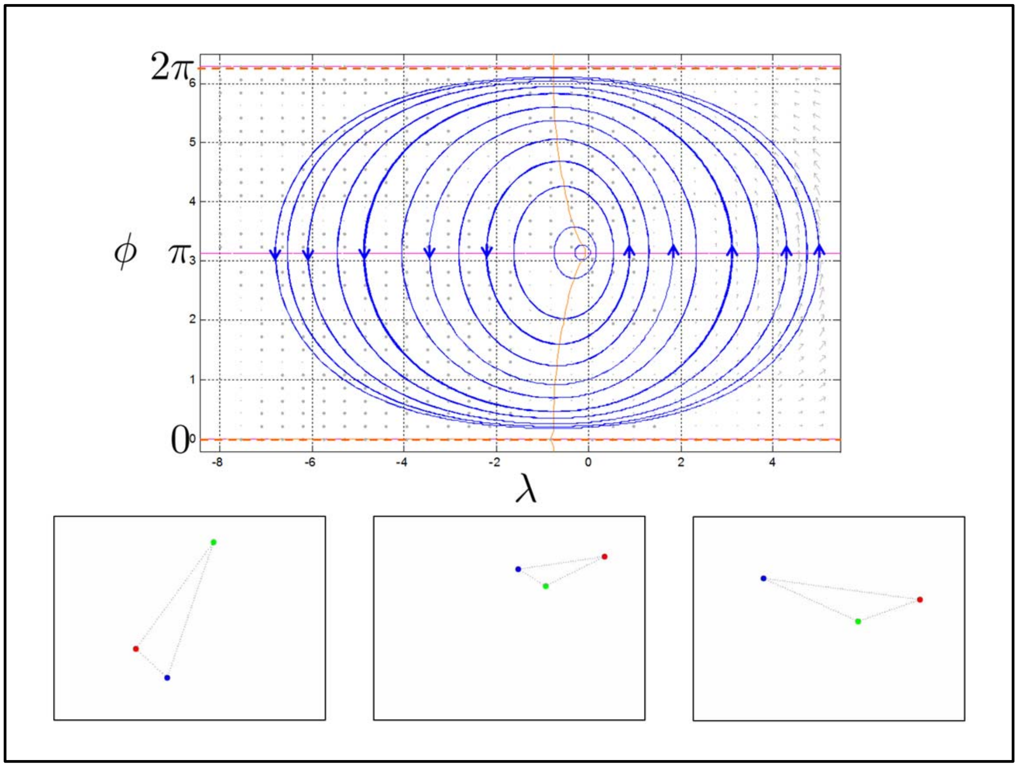
$\alpha = \frac{11\pi}{12}$




$\alpha = \pi$



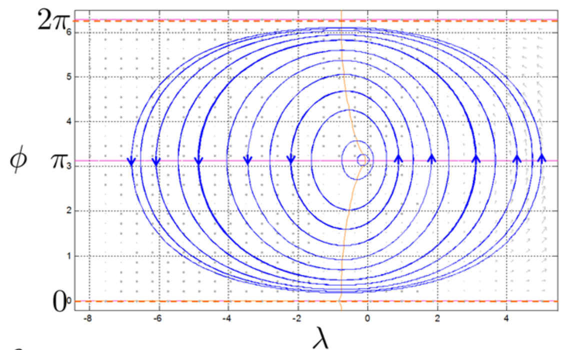
We’ll take a closer look at the pi/2 phase portrait, as well as the corresponding trajectories in the real space, in the next slide


2π
ϕ
π
0
λ
-8
-6
-4
-2
0
2
4
0
1
2
3
4
5
6

Theorem: Let $M \triangleq S^1 \times R - \{\pi, 0\}$, and $M_0 \triangleq \{(\phi, \lambda) \in M | \phi \neq 0\}$.
Every trajectory of the system

$$\phi' = P\left[e^{\lambda}\Big(1 - \cos(\phi)\Big) - 1\right] + \cos(\phi) + e^{\lambda},$$
$$\lambda' = \sin(\phi)\big(Pe^{\lambda} + 1\big)$$

which starts on the set $M - M_0$ is periodic.



Sketch of proof:

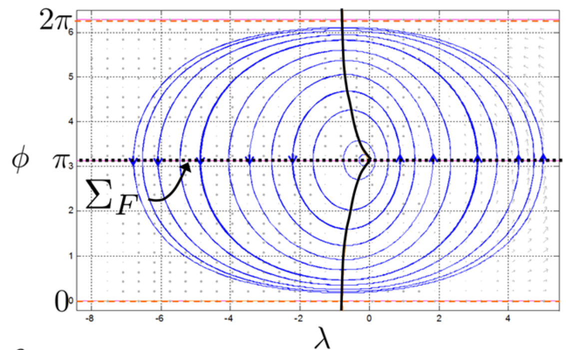
1) Show that the dynamics are *F-reversible* with reverser $F(\phi, \lambda) = (-\phi, \lambda)$, i.e. $F_*(\phi', \lambda') = -(\phi', \lambda')$;

I sketch the main ideas of the proof; make it clear that there's a good bit of analysis in the underlying steps to characterize the regions of the phase portrait, etc.

Theorem: Let $M \triangleq S^1 \times R - \{\pi, 0\}$, and $M_0 \triangleq \{(\phi, \lambda) \in M | \phi \neq 0\}$.
Every trajectory of the system

$$\phi' = P\left[e^{\lambda}\Big(1-\cos(\phi)\Big)-1\right] + \cos(\phi) + e^{\lambda},$$
$$\lambda' = \sin(\phi)\big(Pe^{\lambda}+1\big)$$

which starts on the set $M - M_0$ is periodic.



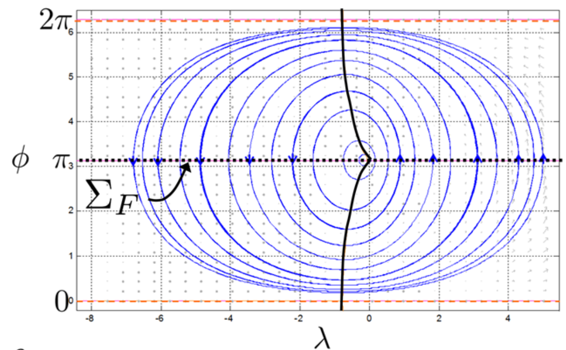
Sketch of proof:

2) Demonstrate that any trajectory which intersects the fixed-point set of F ($\Sigma_F = \{(\phi, \lambda) : \phi = \pi\}$) at one point must intersect Σ_F at another point;

Theorem: Let $M \triangleq S^1 \times R - \{\pi, 0\}$, and $M_0 \triangleq \{(\phi, \lambda) \in M | \phi \neq 0\}$.
Every trajectory of the system

$$\phi' = P\left[e^{\lambda}\left(1 - \cos(\phi)\right) - 1\right] + \cos(\phi) + e^{\lambda},$$
$$\lambda' = \sin(\phi)\left(Pe^{\lambda} + 1\right)$$

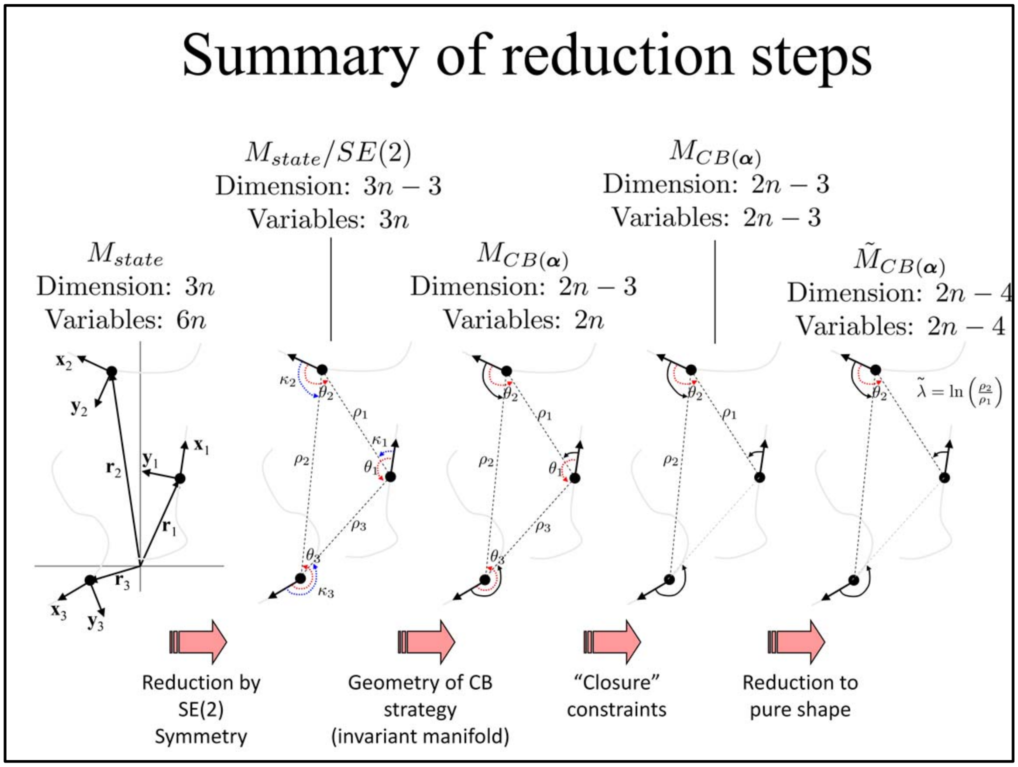
which starts on the set $M - M_0$ is periodic.



Sketch of proof:

3) Invoke Birkhoff Theorem to establish that every trajectory which starts on the set M-M_0 is periodic.

G.D. Birkhoff, "The restricted problem of three bodies," *Rendi. Circ. Mat. Palermo*, Vol. 39, pp. 264-334, 1915.



Reduction to two-dimensional dynamics permits phase portrait analysis

Configuration Space Methods
(saved as back-up for talk)

Data Assimilation

Inverse Problem

Reconstructing collectives from sampled data

The problem of data smoothing and regularized inversion of input data (curvatures, speed, lateral acceleration, jerk)

The SE(2) setting vs. the linear-quadratic optimal control problem for determining jerk.

Regularization of Inverse Problem

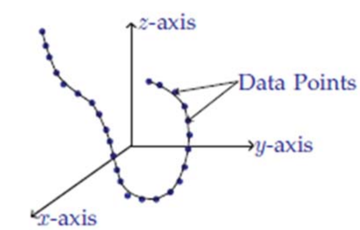
PROBLEM

Given a time series of observed positions $\{r_i\}_{i=0}^{N}$ in three dimensional space, our primary objective is to generate a smooth trajectory to fit these data points.

- A penalty term is introduced to assure smoothness of the reconstructed trajectory.

Fit Error **Penalty Term**

$$\boxed{\sum_{i=0}^{N} \|r(t_i) - r_i\|^2} + \lambda \boxed{\int_{t_0}^{t_N} (\text{Suitable Path Cost})\,dt}$$



- **BUT** the relative importance of the fit error with respect to the penalty term is not known *a priori*.
- The optimal value of the regularization parameter (λ) is chosen using ordinary cross validation, and the optimal value depends on the signal-to-noise ratio in the data.

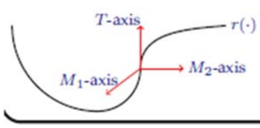
Generative Models

Generative Model I

$$\begin{aligned}\dot{r} &= \nu T \\ \dot{T} &= \nu\left(k_1 M_1 + k_2 M_2\right) \\ \dot{M}_1 &= -\nu k_1 T \\ \dot{M}_2 &= -\nu k_2 T\end{aligned} \qquad (1)$$

- **Penalty Term** to ensure smoothness:

$$\int_{t_0}^{t_N} \left(\dot{k}_1^2 + \dot{k}_2^2 + \dot{\nu}^2\right) dt$$



Model I → Model II

$$\begin{aligned}v &= \nu T \\ a &= \dot{\nu} T + \nu^2 k_1 M_1 + \nu^2 k_2 M_2 \\ u &= (\ddot{\nu} - \nu^3(k_1^2 + k_2^2))T \\ &\quad + (3\nu\dot{\nu} k_1 + \nu^2 \dot{k}_1) M_1 \\ &\quad + (3\nu\dot{\nu} k_2 + \nu^2 \dot{k}_2) M_2\end{aligned}$$

Generative Model II

$$\begin{aligned}\dot{r} &= v \\ \dot{v} &= a \\ \dot{a} &= u\end{aligned} \qquad (2)$$

- **Penalty Term** to ensure smoothness:

$$\int_{t_0}^{t_N} (u^T u)\, dt$$

Model II → Model I

$$\begin{aligned}\nu &= \|v\| \\ T &= \frac{v}{\|v\|} \\ \dot{T} &= \frac{1}{\nu}\left(a - (a \cdot T)T\right) \\ \kappa &= \frac{\|\dot{T}\|}{\nu} \\ \tau &= \frac{v \cdot (a \times u)}{\|v \times a\|^2}\end{aligned}$$

Algorithm

RECONSTRUCTION THROUGH ERROR MINIMIZATION

Model I (Nonlinear)

- Approximation by **piecewise constant** speed and curvature, transforms the problem into a **non-convex numerical optimization problem** [2].
- MATLAB routine: ***fminunc***.
- The algorithm is capable of estimating curvature with higher resolution, but the process is **time consuming**.

Model II (Linear)

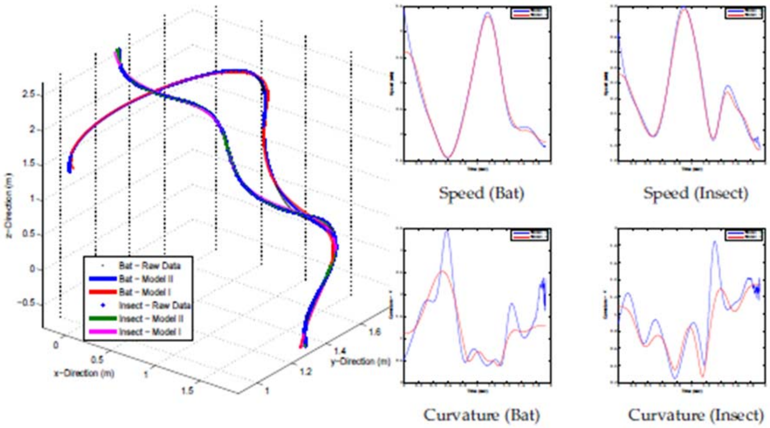
- *Path-independence lemmas* and *Riccatti equation* **ensure global optimality of the solution** and **the solution is semi-analytic** [1].
- Reconstructed positions can be expressed as **linear combinations of raw data**, but the **linear weights vary across data points**.
- This method is **orders of magnitude faster** than the nonlinear version of the story.

B. Dey and P. S. Krishnaprasad (2012), Trajectory smoothing as a linear optimal control problem, in *Proc. 50th Allerton Conference on Communications, Control and Computing*, pp. 1490-1497 (http://dx.doi.org/10.1109/Allerton.2012.6483395).

http://www.isr.umd.edu/Labs/ISL/SMOOTHING/ for documentation and code

Performance

Results (Bat-Mantis Pursuit Event)



ng

Avg. Fit Error	Model I	Model II
Bat	2.2401×10^{-4}	7.6142×10^{-5}
Mantis	1.2966×10^{-4}	2.3913×10^{-5}

Final Remarks

- Geometric viewpoint can be effective in modeling, control, data assimilation and inference of sensorimotor strategies from behavior
- While much of the theoretical discussion here has been focused on 2D, the results for 3D are available

Reference for general framework for collectives (graphs, strategies, feedback laws)

K. S. Galloway, E. W. Justh and P. S. Krishnaprasad (2013), Symmetry and reduction in collectives: cyclic pursuit strategies, *Proc. R. Soc. A* 2013 **469** (http://dx.doi.org/10.1098/rspa.2013.0264).

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