Rigid body attitude stabilization with vector observations

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Outline

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Almost global stabilization

Our aim is to stabilize the rigid body attitude at a given orientation.

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Therefore we cannot design time-invariant feedback control laws which can globally stabilize any given attitude.

The next best thing is to look for almost global stabilization.

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- X has finite number of equilibrium points and the stable manifold of every equilibrium point other than x is a lower dimensional submanifold of M,
- all points in $M \setminus \mathcal{U}$, where $\mathcal{U}$ is the union of stable manifolds of the equilibrium points other than x, converge to x.

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AGAS with external and internal torques

AGAS and vector observations

Summary

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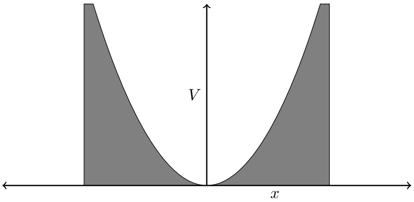

AGAS with external torques

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We shall look for a feedback torque in the lines of the double integrator:

$$u_{ext}(R, \Omega) = C\Omega + \pi(dV(R)),$$

where $C > 0$ and $\pi : T^*SO(3) \longrightarrow \mathbb{R}^3$ is an appropriate map, to achieve AGAS at $R_d \in SO(3)$.

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Notice that the equilibria of the closed loop system are points $(R_c, 0)$ where R_c is a critical point, that is, $dV(R_c) = 0$.

AGAS with external torques (Contd...)

However, there is no smooth potential function on $SO(3)$ with a single critical point.

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Then the feedback torque $u_{ext}(R, \Omega) = C\Omega + \pi(dV(R))$ achieves AGAS at R_d.

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Then the feedback torque $u_{ext}(R, \Omega) = C\Omega + \pi(dV(R))$ achieves AGAS at R_d.

The proof makes use of the fact that the closed loop system can be derived from a Riemannian structure on $SO(3)$ and linearization using these ideas.[2]

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Internal actuation with reaction wheels

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\begin{aligned}
\dot{R} &= R\hat{\Omega}_b, \\
I_L\dot{\Omega}_b + I_r\dot{\Omega}_r &= (I_L\Omega_b + I_r\Omega_r) \times \Omega_b, \\
I_r\dot{\Omega}_b + I_r\dot{\Omega}_r &= u_{int}, \\
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Reaction wheels as 'torque producing devices'

The equations can be rearranged

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where

$$u_{eq} = -u_{int} - \Omega_b \times h.$$

The intrinsic approach

Using the conservation of angular momentum, the equations of motion can be written as

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Similarity with external actuation

Compare this with equations of external actuation

$$\begin{aligned} \dot{R} &= R\hat{\Omega}, \\ I\dot{\Omega} &= I\Omega \times \Omega + u_{ext}. \end{aligned}$$

DOES $u_{int} = -u_{ext}$ ACHIEVE AGAS?

A natural question to ask is whether

$$u_{int}(R, \Omega_b) = -C\Omega_b - \pi(dV(R))$$

can yield AGAS.

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Recall that the case of external actuation was analyzed by linearizing the Riemannian structure.

However, for the closed loop system with internal actuation

$$\begin{aligned} \dot{R} &= R\widehat{\Omega}_b \\ I_s\dot{\Omega}_b &= R^T\mu \times \Omega_b + C\Omega_b + \pi(dV(R)), \end{aligned}$$

it is not clear how to provide such a structure.

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Suppose $\psi : \mathbb{R}^n \longrightarrow M$ is a local parameterization of M around x_0 ($\psi(0) = x_0$). Define

$$Y := \psi_* X,$$

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Linearization of Y

The linearization of Y at 0 can be obtained as follows:

$$DY(0)\,\gamma := \left.\frac{d}{dt}\right|_{t=0} Y(t\gamma).$$

This is equivalent to linearizing X at x_0.

Linearization

The exponential coordinates for $SO(3)$

We define the parameterization from $\psi : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow SO(3) \times \mathbb{R}^3$ as $\psi = (\exp, \mathrm{id})$, where $\exp$ is the usual matrix exponential map.

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For checking AGAS, we linearize at every equilibrium point, to get a linear gyroscopic system with damping

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Similarity with external actuation

If we linearize using a similar procedure the external actuation case, we get

$$I\ddot{\eta} + C\dot{\eta} + \delta^2 \tilde{V}(0)\eta = 0.$$

Comparison of external and internal actuation

When $C > 0$, the stability of both of these systems depends identically on $\delta^2\tilde{V}(0)$ and the behaviour is hyperbolic at every equilibrium point.

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Theorem

Suppose $V : SO(3) \longrightarrow \mathbb{R}$ satisfies the previous assumption. Then,

- *The control law of the form*

$$u_{ext}(R, \Pi) = CI^{-1}\Pi + \pi\left(dV(R)\right)$$

  almost globally stabilizes the equilibrium point $(R_d, 0)$ for the externally actuated system.

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A 'counterexample'

Examples of control laws which do not fall to this class exist which do not achieve AGAS with a change of sign:

$$u(R) = C\Omega - k_1 \times R^T k_2 - k_2 \times R^T k_3 - k_3 \times R^T k_1.$$

locally stabilizes around the identity with internal actuation, but not so for external actuation.

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Construction of V

We are on the lookout for a function V that satisfies the required conditions of AGAS.

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The function $\mathrm{trace} : SO(3) \longrightarrow \mathbb{R}$

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The modified trace function $\mathrm{trm}_P : SO(3) \longrightarrow \mathbb{R}$

$$\mathrm{trm}_P(R) = \mathrm{trace}(PR),$$

where P is a symmetric matrix.[3]

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Modified trace functions (MTFs)

Lemma

If P is a symmetric 3×3 matrix with distinct eigenvalues π_1, π_2, π_3 and if

$$(\pi_1 + \pi_2)(\pi_2 + \pi_3)(\pi_3 + \pi_1) \neq 0,$$

then there are exactly four regular critical points of trm_P.

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It turns out that MTFs are tailor-made for AGAS.

They have been used for almost global stabilization using external actuation by Koditschek.[4]

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The spinning top potential

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We augment V_g as follows

$$\tilde{V}(R) = ce_3 \cdot R^T e_3 + e_1 \cdot R^T e_1.$$

The spinning top potential (Contd...)

It turns out that

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The control torque due to the above potential is

$$\pi(dV(R)) = -e_1 \times R^T e_1 - c e_3 \times R^T e_3.$$

Vector observations

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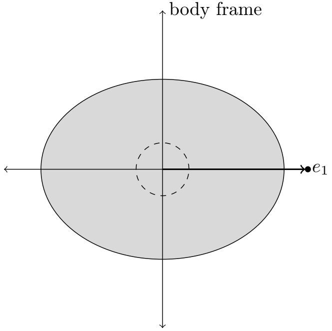

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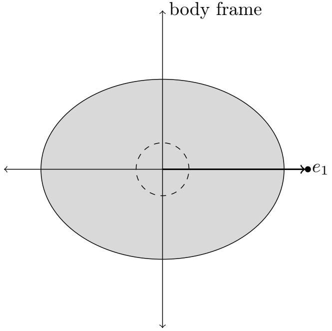

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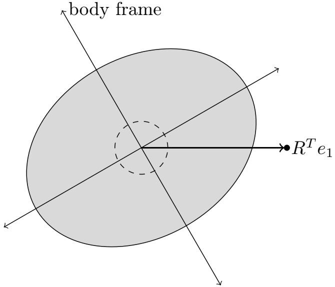

Figure: Present configuration

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AGAS at arbitrary R_d

The above function can be modified to

$$V_{R_d}(R) = R_d^T k_1 \cdot R^T k_1 + R_d^T k_2 \cdot R^T k_2,$$

satisfies the conditions for AGAS at any $R_d \in SO(3)$.

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It turns out that the above function is an MTF too.

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It is tempting to add more vectors to the function for a 'better performance':

$$V(R) = \sum_{i=1}^{m} c_i k_i \cdot R^T k_i.$$

It turns out that the above function is an MTF too.

But it may not satisfy the conditions for AGAS. Presence of multiple k_is makes the analytical verification difficult.

Outline

Summary

There exist a class of control laws which stabilize a rigid body with internal or external actuation with only a change of sign.

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There exist a class of control laws which stabilize a rigid body with internal or external actuation with only a change of sign.

It is possible to construct such control laws using only the vector observations, without having to determine the attitude matrix explicitly.

Thank You